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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LIV. No. 5. MARCH 1894.

Fellows of the Society are informed that post-cards containing the titles of papers to be read at the evening meetings will be sent to those who shall express their wish to receive them. Fellows desiring to receive the post-cards are requested to send their names to the Assistant-Secretary, Mr. W. H. Wesley, Burlington House, London, W.

In order to enable the Secretaries to prepare complete lists in time for the issue of the post-cards, authors are particularly requested to send in their papers not later than the Tuesday preceding the day of meeting.

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VOL. LIV. MARCH 9, 1894. No. 5

Captain W. de W. ABNEY, C.B., R.E., D.C.L., F.R.S., President, in the Chair.

Augustin Stanislaus Ghosh, 28 Convent Road, Calcutta;
Lieutenant William H. F. Montanaro, R.N., H M.S. *Canada*, Bermuda;
Walter A. Post, Newport News, Virginia, U.S.A.;
Alexander William Roberts, Lovedale, Alice, Cape Colony,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers, from personal knowledge, being appended :—

John Briggs, B.A. Cantab., University Tutorial College, Red Lion Square, W.C. (proposed by Dr. R. Bryant);
Rev. Samuel Robert Browne, M.A., LL.D., Mathematical Master in Bristol Grammar School, Camelot, St. John's Road, Clifton (proposed by F. H. Stevens);
Rev. F. J. Eld, M.A. Oxon., Whiteladies, Worcester (proposed by Captain W. de W. Abney);
Henry Owen, F.R.G.S., F.R.Met.Soc., Royal Thames Yacht Club, 7 Albemarle Street, W. (proposed by Capt. R. Reynolds);
George C. Pulsford, Head Master, Royal Hospital School, Greenwich, S.E. (proposed by H. H. Turner);
W. Rickmer Rickmers, Schwarzspanierstrasse, Vienna (proposed by Herbert Ingall);
William Stewart Stewart, Levern House, Barrhead, Scotland (proposed by Adam Hilger).

One hundred and three presents were announced as having been received since the last meeting, including, amongst others:—

Annals of the Astronomical Observatory of Harvard College, vols. xxv., xxix., xxxi. part 2; vol. xl. part 2, presented by the Observatory; A. Auwers, Tobias Mayer's Sternverzeichniss, nach den Beobachtungen auf der Göttinger Sternwarte, 1756–60, presented by Professor Auwers; H. Gyldén, Traité Analytique des orbites absolues des huit planètes principales, tome i., presented by the author; photographs of portions of the Moon, retouched enlargements from negatives taken at the Lick Observatory, presented by Dr. L. Weinek.

Remarks upon Mr. Stone's Proposed Corrections to the Measure of Time since 1864. By Professor Simon Newcomb.

In the December number of the *Monthly Notices* Mr. Stone presents a very able and elaborate statement of certain corrections which he claims should be applied to the adopted measure of time from and after the beginning of 1864, together with the resulting corrections to the tabular places of the Moon derived from Hansen's tables. The great importance of the question raised by Mr. Stone seems to make it only respectful to him that I should once more state the difficulty in the way of accepting his conclusions.

The following numbers are extracted from Mr. Stone's paper on page 4 of the *Monthly Notices* for November, which give the results of certain Radcliffe observations of the Moon, made in 1892:—

Date, 1892 *January* 6.

	m s
Correction to be subtracted from M.T. for change of Sidereal Time at Mean Noon since 1864	41·51
Observed Right Ascension of the Moon	49 19·57
R.A. from Hansen's Tables for uncorrected Mean Times ...	20·93
Hansen minus observed uncorrected	+1·36
Correction due to the change of unit in M.T.	−1·46
Hansen minus observed corrected	−0 10

In this and other papers it is claimed by Mr. Stone that the above correction of $41^{s}\cdot51$ to the sidereal time of mean noon since 1864 has resulted from a progressive change, starting from the epoch 1864·0 and increasing uniformly at the rate of about $1^{s}\cdot48$ annually. From this it follows that the tabular Right Ascensions of the Moon at transit over the meridian at Oxford, as computed from Hansen's Tables, should on the above date

receive a correction of $1^s \cdot 36$ in consequence of the alleged change in the sidereal time at mean noon.

The claim that this correction starts from zero about 1864·0, and increases uniformly at the rate of about $1^s \cdot 48$ annually, is based by Mr. Stone on the fact that that was the date when Le Verrier's tables were introduced into the *Nautical Almanac*, and when, in consequence, according to Mr. Stone's theory, the new measure of time began.

Now, to show my difficulty, let us suppose that the old tables of the Sun had been continued up to the end of the year 1891, and that in the year 1892 Le Verrier's Tables had for the first time been introduced. To make my meaning perfectly clear I shall call the case of actual introduction in 1864, case A; and the supposed case of introduction in the year 1892, case B. On Mr. Stone's theory, the time t should in case B be counted from 1892·0, and the correction therefore for 1892 January 6 would have been substantially zero.

But it is evident that the tabular numbers for the whole year 1892, and therefore for January 6, would have been exactly the same in case B as they are in case A; that is to say, if the solar ephemeris from Le Verrier's Tables had been computed for the first time for the year 1892, all the quantities taken from the tables and printed in the *Nautical Almanac* would have been the same as they actually are. To claim that they would have been different would imply that the results of making a computation from Le Verrier's Tables are not merely a function of the numbers contained in the tables, but of the time when they began to be used for the ephemeris of the *Nautical Almanac*. Were this doctrine true it might also be possible that the quantities taken out from a table of logarithms were different from year to year. As this cannot be the case it conclusively follows that all the numbers used in computing the tabular place of the Moon at transit over the meridian of Oxford on 1892 January 6 would be exactly the same in case B that they are in case A. Consequently the result would have been the same as Mr. Stone gives it for the uncorrected time; that is to say, we should have had—

	m	s
Observed Right Ascension of the Moon	49	19·57
R.A. from Hansen Tables for uncorrected M.T.		20·93
Hansen minus observed uncorrected		+1·36
Correction due to the change of unit in M.T.		0·00
Hansen minus observed corrected		+1·36

Thus Mr. Stone's theory and his method of applying it give a correction zero for 1894·0 in case B, when in fact the numbers to be corrected are the same as they are in case A. From this conclusion I see no escape. From Mr. Stone's theory the use of the new unit of time would, in case B, have commenced in 1894.

Here t must be reckoned from 1894·0, and the correction must be zero. Thus his theory gives two inconsistent corrections to the same numbers.

Note on Professor Newcomb's Remarks. By E. J. Stone, M.A., F.R.S., Radcliffe Observer.

Professor Newcomb's attempt to disprove my views by a *reductio ad absurdum* will be found to rest upon an interesting example of the very common logical fallacy of a *petitio principii*.

I have proved, or at least profess to have proved, in the *Monthly Notices*, vol. liv., that, with the two independent systems of time-measurement adopted in practical astronomical work, the tabular right ascensions, α^s, for the transits over an observer's meridian at the time $r+\delta r$ can only be accurately found from the equation

$$(1) \qquad \alpha^s = \phi\,(r+\delta r) = \phi\,(r) + \frac{d\,.\,\phi\,(r)}{dt}\left[\frac{\alpha^s - A_r{}^{s\prime} - \delta l^s - 86400^s\,.\,xr}{24^h\left(1+\frac{N}{2\pi}+x\right)}\right] + \&c.,$$

where $\phi(r)$ is the tabular right ascension computed directly from the tables for the time $t=r$; $A_r{}^{s\prime}$ the "local sidereal time at mean noon," computed for the time $t=r$, from a set of solar tables adopted for this purpose, and also used to fix the time at which in each year the Sun's mean longitude is equal to 280°; the corrections δl^s and x are quantities whose magnitudes depend upon the differences between the constants l and N, adopted for the epoch constant and mean motion of the Sun per "unit day" in the formation of the solar tables employed, and certain definite constants l_0 and N_0, which, if they had been adopted instead of l and N, would have rendered the corrections δl^s and x zero. These corrections, δl^s and x, necessarily change with the adoption of different values of l and N in our solar tables.

Now Professor Newcomb, undoubtedly following the usual course, determines certain quantities $\alpha_l{}^s$ from the equation

$$(2) \qquad \alpha_l{}^s = \phi\,(r) + \frac{d\,.\,\phi\,(r)}{dt}\left[\frac{\alpha_l{}^s - A_r{}^{s\prime}}{24^h\left(1+\frac{N}{2\pi}\right)}\right] + \&c.,$$

and calls the results thus obtained the tabular right ascensions at the meridian transits.

But if the equation (1) is exact, the equation (2) is inexact, and *vice versâ*.

It is therefore simply begging the question to assume, without proof, that the values of $\alpha_l{}^s$ found from the equation (2) are the correct tabular right ascensions at the meridian transits, and not the values α^s found from the equation (1).

If Professor Newcomb can *prove* that with the dual systems of time-measurement in use in practical astronomical work the equation (1) is inexact and (2) is exact, then there is an end of my explanation of the cause of the change of the residual differences (Tabular R.A. from Hansen—Observed R.A. of Moon) about the year 1864, which is evident when the quantities $a_{/}^{s}$ computed from the equation (2) are adopted as the tabular right ascensions for the meridian transits. But, on the other hand, if my proofs of the equation (1) are sound, the explanation of this *per saltum* change is shown conclusively to result from the differences between the quantities $a^{s} - a_{/}^{s}$ before and after 1864, which directly result from the different values of the constants l and N employed in the formation of the solar tables in use before and after 1864.

The discussion of the residuals (Hansen—Observed) is not appealed to as a proof of the truth of the equation (1), which must be, and has been, independently proved; but it is certainly no disproof to show that if we neglect the corrections $\delta l^{s} + 86400^{s} \cdot .xr$ and use the quantities $a_{/}^{s}$ for the tabular right ascensions at the meridian transits of the Moon, the residuals which should remain sensibly unchanged about 1864, if the neglect of these corrections is a legitimate one, are found to have changed to such an extent that the same mean motions will not represent the observations before and after that date, and that these *per saltum* changes disappear when the tabular right ascensions are computed from equation (1).

Tables to Facilitate the Application of Gauss's Method of Computing Secular Variations. By R. T. A. Innes.

In Vol. I. of Newcomb's Astronomical Papers of the American Ephemeris, Professor G. W. Hill suggested the computations of the functions $\chi(\tau)$ and $\psi(\tau)$, both functions of θ.

If we take another function α related to θ as follows:—

$$\frac{1-\alpha}{1+\alpha} = \sqrt{\frac{\cos\left(60^{\circ} + \frac{\theta}{3}\right)}{\cos\frac{\theta}{3}}}.$$

M. O. Callandreau has published tables to six places of $\log m^{4} n^{4} \chi(\tau)$ and $\log \frac{\psi(\tau)}{\chi(\tau)}$ in vol. xviii of *Les Annales de l'Observatoire de Paris* for 580 values of α from 0·000 to 1·000, its extreme limits.

I had, previously to seeing M. Callandreau's tables, com-

menced calculating log M and log N, using θ as the argument. They are connected with Hill's functions as follows :—

$$M = \left[\frac{\cos\left(60^\circ - \frac{\theta}{3}\right)}{\cos\theta} \right]^{-2} \psi(\tau)$$

$$N = \frac{\psi\tau}{\chi\tau}.$$

I believe such tables of log M and log N will be useful to astronomers as a check to M. Callandreau's. They also avoid the calculation of the intermediate function a.

Most of the values of log M and log N given in the tables herewith were computed with seven-figure logarithms, so that the last figure is uncertain. I have arranged the tables in the form—

$$f(\theta + h) = f(\theta) + h\frac{df}{d\theta} + h^2\frac{1}{2}\frac{d^2f}{d\theta_2}.$$

Thus to compute log N for $-52^\circ\ 32'\ 36''$, which we may write $-52^\circ - {}^\circ{\cdot}5433$ or $-53^\circ + {}^\circ{\cdot}4567$

-52°	gives	+0·1381654		-53°	gives	+0·1385790
$-{\cdot}5433$	,,	+ 2278		$+{\cdot}4567$	,,	− 1864
$+{\cdot}295$	,,	− 16		$+{\cdot}21$	,,	− 11
		+0·1383916				+0·1383915

The differences from 60° to 90° of θ are not given; they will be required but rarely, and M. Callandreau's tables are to be preferred for high values.

The numerical application of Hill's formulæ suggests a few modifications.

We may divide A, B, and C by a^2; the value of θ will be unchanged. V may be equal to

$$p + q\frac{\psi(\tau)}{\chi(\tau)}.$$

Then q may be removed from F_1, F_2, and F_3.

In F_1 and J_1 write $\left(\frac{a'}{a}\right)^2$ or a^2 for $\frac{1}{a^2}$, a being the disturbing planet. From F_3, J_3, and $\frac{a}{\tau}W_0$ the factor sin I may be removed to the final equations. From $\frac{a}{\tau}R_0$, $\frac{a}{\tau}S_0$, and $\frac{a}{\tau}W_0$ the factor

$$\left(\frac{a_1}{a}\right)^2 \cos^2\phi$$

may be likewise removed to the end.

We have now

$$\frac{a}{r}\mathrm{R}_0 = \left(\frac{r}{a}\right)q^{-\frac{7}{2}}\left[\frac{\cos\left(60^\circ - \frac{\theta}{3}\right)}{\cos\theta}\right]^2 \mathrm{M}(\mathrm{F}_1 + \mathrm{J}_1\mathrm{V})$$

with corresponding equations for $\frac{a}{r}\mathrm{S}_0$ and $\frac{a}{r}\mathrm{W}_0$.

Not the least merit of Gauss's methods consists in its numerous checks. Besides the even and odd functions agreeing in the totals, as pointed out by Professor Hill, we get from

$$\left[\frac{da}{dt}\right] = 0$$

the valuable check

$$\mathrm{B}_0^{(c)} = -\frac{1}{2}\mathrm{A}_1^{(s)}\chi\tan\phi.$$

Also as A, B sin ϵ, B cos ϵ, J_1, &c., can be made explicit functions of sin E, cos E, &c., the sum total of values can be ascertained at the beginning, and will prevent systematic errors. One example will suffice :—

$$\mathrm{B}\sin\epsilon = -\left[k_1\frac{a_1}{a}e\cos\phi_1\sin\mathrm{K}_1\right] + \mathrm{C}_1\cos\mathrm{E} + \mathrm{C}_2\sin\mathrm{E}_1;$$

and as the sums of cosines and sines for any even regular division of the circumference is nil, if we take n divisions, the sum total of B sin ϵ will be

$$-n\left[k_1\frac{a_1}{a}e\cos\phi_1\sin\mathrm{K}_1\right].$$

Gauss's method will also give the perturbations independent of the position of the disturbing planet. In the case of the Earth disturbed by *Venus* we have approximately

$$\frac{d\phi}{dt} = \frac{m_1 n}{1+m}\left(\frac{a_1}{a}\right)^2\cos^2\phi_1\cos\phi\sin\mathrm{E}\,.\,\frac{a}{r}\mathrm{R}_0;$$

$$\int\frac{d\phi}{dt}\,dt = \frac{-m}{1+m}\left(\frac{a_1}{a}\right)^2\sin 1''\,\mathrm{A}_3^{(c)}\sin(l''-\varpi'');$$

$$\delta\phi = 0''{\cdot}908\sin(l''-\varpi''). \qquad [\text{Le Verrier} = 0''{\cdot}907.]$$

θ	Log M	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$	Log N	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$
−90°	9·5573194	− 0	− 38	0·1461280	− 0	−55
−89	9·5573156	− 77	− 38	0·1461225	− 110	−55
−88	9·5573041	− 153	− 38	0·1461060	− 220	−55
−87	9·5572849	− 230	− 38	0·1460784	− 331	−55
−86	9·5572581	− 306	− 38	0·1460398	− 441	−55
−85	9·5572237	− 383	− 38	0·1459902	− 551	−55
−84	9·5571816	− 460	− 38	0·1459296	− 662	−55
−83	9·5571318	− 536	− 38	0·1458579	− 772	−55
−82	9·5570743	− 613	− 38	0·1457752	− 882	−55
−81	9·5570092	− 690	− 38	0·1456815	− 992	−55
−80	9·5569363	− 767	− 38	0·1455768	− 1102	−55
−79	9·5568559	− 843	− 38	0·1454610	− 1213	−55
−78	9·5567677	− 920	− 39	0·1453342	− 1323	−55
−77	9·5566718	− 997	− 39	0·1451964	− 1433	−55
−76	9·5565682	− 1074	− 39	0·1450476	− 1544	−55
−75	9·5564569	− 1152	− 39	0·1448877	− 1654	−55
−74	9·5563378	− 1229	− 39	0·1447168	− 1764	−55
−73	9·5562111	− 1306	− 39	0·1445348	− 1875	−55
−72	9·5560766	− 1384	− 39	0·1443419	− 1985	−55
−71	9·5559343	− 1461	− 39	0·1441379	− 2095	−55
−70	9·5557843	− 1539	− 39	0·1439228	− 2205	−55
−69	9·5556265	− 1617	− 39	0·1436968	− 2316	−55
−68	9·5554609	− 1695	− 39	0·1434597	− 2426	−55
−67	9·5552875	− 1773	− 39	0·1432116	− 2536	−55
−66	9·5551063	− 1851	− 39	0·1429524	− 2647	−55
−65	9·5549173	− 1929	− 39	0·1426822	− 2757	−55
−64	9·5547204	− 2008	− 39	0·1424010	− 2867	−55
−63	9·5545157	− 2087	− 39	0·1421087	− 2978	−55
−62	9·5543031	− 2165	− 39	0·1418054	− 3088	−55
−61	9·5540826	− 2244	− 40	0·1414911	− 3199	−55
−60	9·5538542	− 2324	− 40	0·1411657	− 3309	−55
−59	9·5536179	− 2403	− 40	0·1408293	− 3419	−55
−58	9·5533736	− 2483	− 40	0·1404818	− 3530	−55
−57	9·553121	− 2562	− 40	0·1401233	− 3640	−55
−56	9·5528612	− 2642	− 40	0·1397538	− 3750	−55
−55	9·5525929	− 2722	− 40	0·1393733	− 3861	−55
−54	9·5523167	− 2803	− 40	0·1389817	− 3971	−55
−53	9·5520324	− 2883	− 40	0·1385790	− 4082	−55

θ	Log M	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$	Log N	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$
°						
−52	9·5517400	− 2964	− 40	0·1381654	− 4192	−55
−51	9·5514395	− 3045	− 40	0·1377407	− 4302	−55
−50	9·5511309	− 3127	− 41	0·1373049	− 4413	−55
−49	9·5508141	− 3208	− 41	0·1368582	− 4523	−55
−48	9·5504892	− 3290	− 41	0·1364004	− 4633	−55
−47	9·5501561	− 3372	− 41	0·1359315	− 4744	−55
−46	9·5498147	− 3455	− 41	0·1354517	− 4854	−55
−45	9·5494651	− 3537	− 41	0·1349608	− 4964	−55
−44	9·5491072	− 3620	− 42	0·1344588	− 5074	−55
−43	9·5487410	− 3703	− 42	0·1339459	− 5184	−55
−42	9·5483665	− 3787	− 42	0·1334220	− 5295	−55
−41	9·5479834	− 3871	− 42	0·1328870	− 5405	−55
−40	9·5475922	− 3956	− 42	0·1323410	− 5515	−55
−39	9·5471924	− 4040	− 42	0·1317840	− 5625	−55
−38	9·5467841	− 4125	− 42	0·1312160	− 5735	−55
−37	9·5463674	− 4210	− 43	0·1306369	− 5845	−55
−36	9·5459420	− 4296	− 43	0·1300469	− 5955	−55
−35	9·5455081	− 4382	− 43	0·1294459	− 6065	−55
−34	9·5450656	− 4468	− 43	0·1288339	− 6175	−55
−33	9·5446144	− 4555	− 44	0·1282109	− 6284	−55
−32	9·5441545	− 4643	− 44	0·1275770	− 6394	−55
−31	9·5436858	− 4730	− 44	0·1269321	− 6504	−55
−30	9·5432084	− 4818	− 44	0·1262762	− 6614	−55
−29	9·5427221	− 4907	− 44	0·1256093	− 6723	−55
−28	9·5422270	− 4996	− 44	0·1249316	− 6832	−55
−27	9·5417229	− 5085	− 45	0·1242429	− 6941	−55
−26	9·5412099	− 5175	− 45	0·1235433	− 7051	−55
−25	9·5406879	− 5266	− 45	0·1228327	− 7160	−55
−24	9·5401568	− 5356	− 46	0·1221113	− 7269	−54
−23	9·5396166	− 5448	− 46	0·1213790	− 7378	−54
−22	9·5390673	− 5540	− 46	0·1206357	− 7487	−54
−21	9·5385087	− 5632	− 46	0·1198817	− 7595	−54
−20	9·5379409	− 5725	− 46	0·1191167	− 7704	−54
−19	9·5373638	− 5818	− 47	0·1183410	− 7812	−54
−18	9·5367773	− 5912	− 47	0·1175544	− 7920	−54
−17	9·5361814	− 6007	− 47	0·1167571	− 8027	−54
−16	9·5355760	− 6102	− 48	0·1159490	− 8135	−54
−15	9·5349610	− 6197	− 48	0·1151301	− 8243	−54

θ	Log M	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$	Log N	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$
−14	9·5343364	− 6293	− 48	0·1143005	− 8350	−54
−13	9·5337022	− 6390	− 49	0·1134601	− 8457	−54
−12	9·5330583	− 6488	− 49	0·1126091	− 8564	−53
−11	9·5324045	− 6586	− 49	0·1117474	− 8670	−53
−10	9·5317410	− 6686	− 50	0·1108751	− 8776	−53
− 9	9·5310674	− 6785	− 50	0·1099922	− 8882	−53
− 8	9·5303839	− 6885	− 51	0·1090987	− 8988	−53
− 7	9·5296902	− 6986	− 51	0·1081946	− 9093	−52
− 6	9·5289865	− 7088	− 51	0·1072800	− 9198	−52
− 5	9·5282726	− 7191	− 52	0·1063549	− 9303	−52
− 4	9·5275483	− 7294	− 52	0·1054195	− 9407	−52
− 3	9·5268137	− 7399	− 52	0·1044736	− 9510	−52
− 2	9·5260686	− 7504	− 53	0·1035174	− 9614	−52
− 1	9·5253130	− 7609	− 53	0·1025508	− 9717	−52
0	9·5245468	− 7716	− 53	0·1015739	− 9820	−51
+ 1	9·5237699	− 7823	− 54	0·1005869	− 9922	−51
+ 2	9·5229823	− 7932	− 54	0·0995895	−10024	−51
+ 3	9·5221837	− 8040	− 55	0·0985821	−10124	−50
+ 4	9·5213742	− 8151	− 55	0·0975647	−10224	−50
+ 5	9·5205536	− 8262	− 56	0·0965374	−10324	−50
+ 6	9·5197219	− 8373	− 56	0·0954999	−10424	−50
+ 7	9·5188789	− 8487	− 57	0·0944526	−10522	−49
+ 8	9·5180245	− 8601	− 57	0·0933955	−10620	−49
+ 9	9·5171588	− 8715	− 58	0·0923286	−10717	−48
+10	9·5162815	− 8831	− 58	0·0912521	−10813	−48
+11	9·5153925	− 8949	− 59	0·0901659	−10909	−48
+12	9·5144917	− 9067	− 59	0·0890704	−11004	−47
+13	9·5135791	− 9186	− 60	0·0879652	−11098	−47
+14	9·5126545	− 9307	− 61	0·0868508	−11190	−46
+15	9·5117177	− 9429	− 61	0·0857271	−11283	−46
+16	9·5107687	− 9552	− 62	0·0845942	−11374	−45
+17	9·5098073	− 9676	− 62	0·0834523	−11464	−45
+18	9·5088334	− 9802	− 63	0·0823014	−11553	−44
+19	9·5078469	− 9929	− 64	0·0811418	−11640	−44
+20	9·5068476	−10057	− 64	0·0799734	−11727	−43
+21	9·5058355	−10186	− 65	0·0787964	−11813	−43
+22	9·5048104	−10318	− 66	0·0776110	−11896	−42
+23	9·5037720	−10450	− 66	0·0764173	−11979	−41

θ	Log M	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$	Log N	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$
+24°	9·5027204	−10584	− 67	0·0752152	−12060	−40
+25	9·5016552	−10720	− 68	0·0740053	−12139	−39
+26	9·5005764	−10857	− 69	0·0727875	−12217	−39
+27	9·4994838	−10996	− 70	0·0715619	−12293	−38
+28	9·4983773	−11136	− 71	0·0703288	−12368	−37
+29	9·4972565	−11279	− 71	0·0690884	−12441	−36
+30	9·4961215	−11422	− 72	0·0678407	−12511	−35
+31	9·4949721	−11568	− 73	0·0665861	−12580	−34
+32	9·4938080	−11716	− 74	0·0653247	−12647	−33
+33	9·4926290	−11865	− 75	0·0640567	−12711	−32
+34	9·4914349	−12016	− 76	0·0627825	−12773	−30
+35	9·4902257	−12170	− 77	0·0615022	−12833	−29
+36	9·4890009	−12327	− 78	0·0602160	−12889	−28
+37	9·4877604	−12484	− 79	0·0589243	−12944	−27
+38	9·4865041	−12643	− 80	0·0576272	−12996	−25
+39	9·4852317	−12806	− 82	0·0563251	−13044	−24
+40	9·4839429	−12970	− 83	0·0550183	−13090	−22
+41	9·4826377	−13137	− 84	0·0537070	−13133	−20
+42	9·4813155	−13307	− 86	0·0523918	−13171	−19
+43	9·4799762	−13479	− 87	0·0510728	−13208	−17
+44	9·4786196	−13654	− 88	0·0497504	−13239	−15
+45	9·4772454	−13832	− 90	0·0484251	−13266	−13
+46	9·4758533	−14012	− 91	0·0470972	−13290	−11
+47	9·4744430	−14196	− 92	0·0457671	−13309	− 9
+48	9·4730142	−14382	− 94	0·0444354	−13324	− 6
+49	9·4715666	−14572	− 96	0·0431023	−13334	− 4
+50	9·4700998	−14765	− 97	0·0417686	−13338	− 1
+51	9·4686136	−14961	− 99	0·0404348	−13339	+ 2
+52	9·4671077	−15160	−101	0·0391009	−13332	+ 4
+53	9·4655815	−15365	−103	0·0377683	−13319	+ 7
+54	9·4640347	−15572	−105	0·0364371	−13302	+11
+55	9·4624671	−15783	−107	0·0351078	−13278	+14
+56	9·4608781	−15998	−109	0·0337816	−13246	+18
+57	9·4592675	−16217	−111	0·0324587	−13208	+21
+58	9·4576346	−16442	−113	0·0311401	−13162	+25
+59	9·4559793	−16669	−116	0·0298263	−13107	+30
+60	9·4553008	−16904	−118	0·0285187	−13043	+34

θ	Log M	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$	Log N	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$
+61°	9·4525986	...	...	0·0272177		
+62	9·4508724	...	...	0·0259244		
+63	9·4491215	...	...	0·0246398		
+64	9·4473455	...	...	0·0233648		
+65	9·4455436	...	...	0·0221007		
+66	9·4437154	...	...	0·0208485		
+67	9·4418600	...	...	0·0196097		
+68	9·4399769	...	...	0·0182853		
+69	9·4380653	...	...	0·0171772		
+70	9·4361245	...	...	0·0159863		
+71	9·4341535	...	...	0·0148149		
+72	9·4321517	...	...	0·0136632		
+73	9·4301180	...	...	0·0125364		
+74	9·4280515	...	...	0·0114334		
+75	9·4259511	...	...	0·0103575		
+76	9·4238157	...	...	0·0093111		
+77	9·4216442	+	...	0·0082976		
+78	9·4194350	...	...	0·0073169		
+79	9·4171869	...	...	0·0063752		
+80	9·4148983	...	...	0·0054748		
+81	9·4125674	...	...	0·0046197		
+82	9·4101922	...	...	0·0038142		
+83	9·4077706	...	...	0·0030629		
+84	9·4053004	...	...	0·0023717		
+85	9·4027783	...	...	0·0017468		
+86	9·4002011	...	...	0·0011964		
+87	9·3975647	...	...	0·0007299		
+88	9·3948632	...	...	0·0003601		
+89	9·3920884	...	...	0·0001062		
+90	9·3892283	...	...	0·0000000		

Sydney: 1893 *June* 19.

On the Rotation of Saturn. By A. Stanley Williams.

Introductory.

The rotation of *Saturn* was discovered in 1793 by Herschel from observations of variations in the appearance of his well-known quintuple belt in the southern hemisphere of the planet. Herschel concluded that the rotation was performed in a period of $10^h\ 16^m\ 0^s{\cdot}4$ of mean time, and that this period "must be exact to much less than two minutes either way." * The observations extend over a period of 154 rotations.

An interval of no less than eighty-three years then occurred without any further addition being made to our knowledge of this important quantity. At length, in 1876 December, a bright spot was detected by Professor Asaph Hall, with the Washington 26-inch refractor, near the planet's equator. This spot was two or three seconds in diameter, round, and well defined, and of a brilliant white colour. From a careful discussion of the observations of this spot by six different observers, Professor Hall determined the period of rotation to be $10^h\ 14^m\ 23^s{\cdot}8 \pm 2^s{\cdot}30$.†

In the spring of 1891 a number of bright spots were detected by the writer in the bright equatorial zone of *Saturn*, south of the planet's equator. The observations of these spots have not yet been completely discussed; but, according to Mr. Marth, the motion of five spots (four bright and one dark), of which the observations have been published,‡ is approximately represented by a period of $10^h\ 14^m\ 21^s{\cdot}84$.§

A great many observations of these bright equatorial spots were obtained during the opposition of 1892. The spots themselves were also seen by two or three other observers. These observations have not yet been discussed, but they are probably represented approximately by the rotation period of $10^h\ 13^m\ 38^s{\cdot}4$ adopted by Mr. Marth in his Ephemeris for 1893.‖

The foregoing is a summary of our knowledge respecting the rotation of *Saturn* up to the present year. The remainder of this paper is devoted to a discussion of the results obtained during the opposition of 1893. Most of the observations were made by the writer, but satisfactory results were also obtained by six additional observers, besides several others who saw the spots, but have not given sufficient particulars to enable them to be identified.

* *Philosophical Transactions*, 1794, p. 66.

† *Ast. Nach.*, No. 2146. During the observations sixty-one rotations were performed by the planet.

‡ *Ast. Nach.*, No. 3051.

§ *Monthly Notices*, vol. li. p. 568. Mr. Denning from the observations of ten bright equatorial spots, but employing only the extreme observations of each spot and an approximate method of reduction, makes the rotation period $10^h\ 14^m\ 26^s{\cdot}6$ (see *Observatory*, 1891, p. 369).

‖ *Monthly Notices*, vol. liii. p. 153.

Methods of Observation.

The observations here given, excepting those attributed to other observers, were all made with a $6\frac{1}{2}$-inch reflector by Calver, a power of 225 being generally used, and occasionally 320 when the weather permitted. Three principal methods were employed to determine the positions of spots, namely—

1. *Direct transits.*—In these the time at which a spot seemed to be in mid-transit, or bisected by the central meridian of the disc, was recorded as exactly as possible. This is the method usually employed in the case of the planet *Jupiter*, and it is capable of giving very satisfactory results with this planet in the hands of a careful observer. But with *Saturn* it is not enough to rely wholly upon these direct transits. The spots of *Saturn* are generally so delicate that on an average night it not unfrequently happens that, owing to an interval of unsteady definition, it may not be practicable to see them distinctly for sometimes ten minutes or even more at a time. And the occurrence of such a period of poor seeing at the critical time results in many a direct transit being lost. In order to make up for such losses, and to further increase the accuracy with which the times of the passages over the central meridian of the disc may be fixed, I have therefore largely had recourse to the following:—

2. *Estimated transits.*—An observer systematically devoting himself to the observation of the spots of *Saturn* or of *Jupiter* soon becomes acquainted with the rate at which such spots appear to move across the disc through the rotation of the planet. If, then, a spot be seen at a short distance preceding or following the central meridian of the disc, it is easy for such an observer to estimate the time of the actual transit; and that, too, with considerable accuracy, provided that the object observed is not distant much more than twenty minutes from the transit, either way. Such an estimation forms what is here called an *estimated transit*, an illustration of which will be given later on. This method has one great advantage when observations of such delicate objects as the spots on *Saturn* are in question; and that is, that it is not necessary to wait for some definite moment, which may be lost owing to an interval of poor definition, but at any instant that a clear view of a spot is obtained an estimated transit may be made.

3. A third method consists in recording the time when a spot transits, not over the central meridian of the disc, but over some other meridian, such as the meridian distant thirty degrees from the central meridian. This method is not, however, susceptible of great exactness, and it was only employed in a few exceptional cases.

A direct transit and at least three or four estimated transits of a spot were made on each night whenever the circumstances permitted of so doing. The mean of such estimated transits has

been considered to be of the same value as a direct transit, and the simple mean of the two adopted as the concluded time of transit of the spot. The following example, extracted from my observation book, will serve to illustrate this:—

1893 March 21, $12^h\ 25^m$ (clock time): Dark northern spot β transited. Two e S, m D, pretty well-defined spots on the two chief belts in the northern hemisphere. These two form the most conspicuous and definite dark spots which I have ever seen on these belts. They are quite plain and easy. Estimated transits as under:—

h	m	m	h	m	
12	9 +	12 =	12	21.	225, seen plainly.
12	16 +	7 =	12	23	,, well seen.
12	36 −	17 =	12	19	,,
12	44 −	23 =	12	21	,, definition very confused.

Mean of estimated transits	= 12 21	
Direct transit	= 12 25	
Mean	= 12 23	
Clock correction	= −3·5	
Concluded time of transit	= 12 19·5	G.M.T.

The first of the above estimated transits signifies that at $12^h\ 9^m$ the spot was estimated to be 12^m before transit; the last, that at $12^h\ 44^m$ it was 23^m past transit.

Great pains were taken to avoid any possibility of bias from recollection of previous observations. The experience of preceding years had shown that this is most important, and that errors amounting to as much as a quarter of an hour in the time of a transit may sometimes be made if the observer has some preconceived idea, perhaps erroneous, of the time when a spot ought to transit, from recollection of the work of some preceding night. Accordingly, in the present series of observations, great efforts were made to avoid all recollection of previous results. The times were just recorded in the observation-book, and then intentionally forgotten. This succeeded so well that the whole of the observations of 1893 are absolutely independent, and unaffected by any possibility of bias. This, however, it should be remarked, applies only to the results of different nights; as when a number of estimated transits of a spot were made upon the same night it was impossible sometimes in the later transits to avoid all recollection of the earlier ones, however hard one might try.

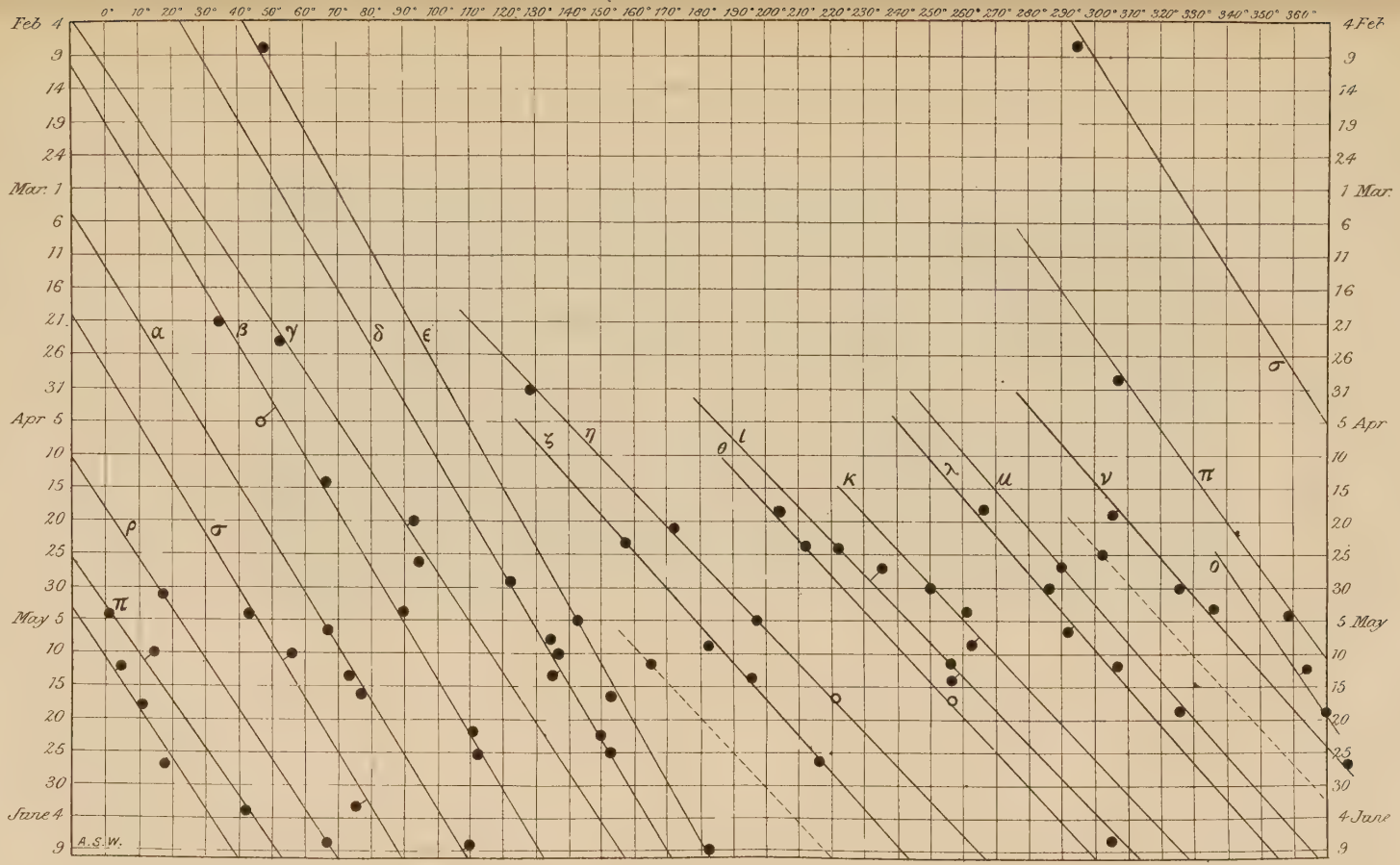
Determination of the Period of Rotation.

Great care was also taken in the final reduction of the observations to ensure that the results were free from any possibility of their being influenced by preconceived ideas. Whilst the work of observation was in progress, all reference to the re-

corded observations was purposely avoided, and it was not until the month of June that the reductions were commenced. The whole of the observations were then reduced, and the concluded times of transit determined in the manner indicated above before any attempt was made at comparing them. The observations were also weighted in as uniform a manner as possible; that is to say, they were weighted partly according to the degree of satisfaction given by them at the time of observation and partly according to the number of observations. All the weights were also finally fixed before any attempt was made at comparison of the different observations, and they have not in any case been subsequently altered, with the single exception that an observation of spot *o* has been reduced from weight 1 to weight $\frac{1}{2}$. A weight $\frac{1}{2}$ signifies that the observations are very rough or few in number. In no case has the concluded time of transit of a spot been subsequently altered. The observations having all been uniformly reduced in this manner, the corresponding longitudes were calculated from Mr. Marth's Ephemeris, and the results laid down upon a diagram or chart (Plate 5) in which the horizontal lines represent dates, and the vertical lines longitudes; the black dots represent the writer's observations, those of other observers being represented by circles. The connection between the different observations could then be readily seen, and lines were drawn through the observations of the same spots.

The method adopted in order to determine the period of rotation from these observations is as follows. In his 'Ephemerides of the Five Inner Satellites of Saturn,' 1893,* Mr. Marth has given a list of the times when an adopted zero meridian would pass the middle of the illuminated disc of *Saturn*. These times are based upon a period of rotation of $10^h\ 13^m{\cdot}64$, and they have already received the necessary corrections for the varying positions of the Earth and *Saturn*, for the light equation, and for phase. It is consequently an easy matter to determine the corrections to be applied to the above period from the observations. Thus, if t be the observed interval in time by which a spot follows the assumed zero meridian at one observation, and t' the corresponding interval at another observation, then the difference between t and t' divided by the number of rotations of the planet in the interval between the two observations will give the correction to be applied to the rotation period of the Ephemeris ($10^h\ 13^m{\cdot}64$). An example will make this clear. Spot β followed the zero meridian of the Ephemeris on March 21 by an interval of $0^h\ 58^m{\cdot}1$. On May 25 this interval was $3^h\ 10^m{\cdot}4$. The difference between the two amounts to $132^m{\cdot}3$. In the interval between these two observations the planet completed 152 rotations; so that we have $\frac{132{\cdot}3}{152}=0^m{\cdot}870$ as the correction to be applied to the period of the Ephemeris from the comparison

* *Monthly Notices*, vol. liii. p. 153.


0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110° 120° 130° 140° 150° 160° 170° 180° 190° 200° 210° 220° 230° 240° 250° 260° 270° 280° 290° 300° 310° 320° 330° 340° 350° 360°
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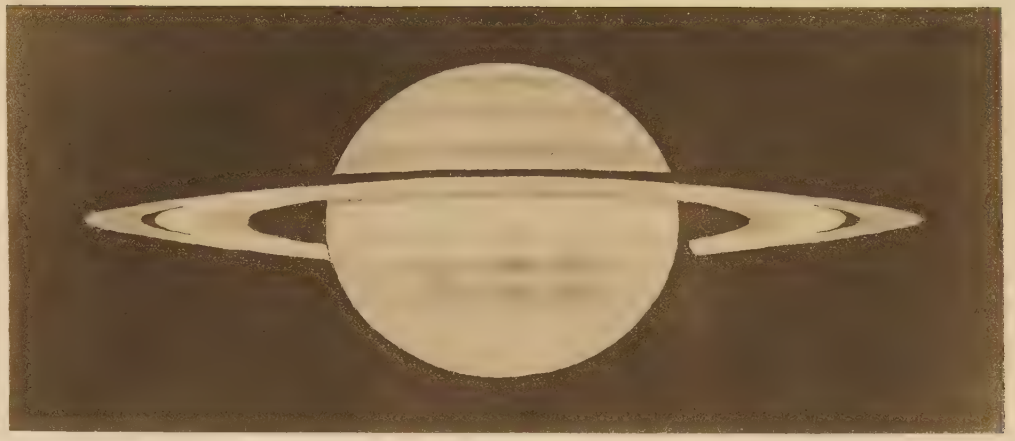


A. Stanley Williams, Del.

SATURN.

GENERAL ASPECT IN THE SPRING OF 1893.

WOODBURY COMPANY (EYRE AND SPOTTISWOODE), LONDON.

between these two observations, which correction will be additive.

The operation here performed has been repeated, so that each observation has been compared with all the other observations of the same spot, and the simple mean of the comparisons of one observation with all the others has been considered as the period of rotation given by such observation, and to have a corresponding weight; and the weighted mean of all the different results is the concluded period of rotation of the spot.

The nature of the tables which follow will now be readily comprehended. The first four columns explain themselves. Column five contains the observed times of transit reduced to some specified epoch by means of the concluded period of rotation, and column six the differences from the mean of such reduced observations. In the seventh column will be found the rotation periods resulting from the comparison of each observation with all the others.* The following abbreviations are used in the descriptive notes: S=small, v S=very small, e S=exceedingly small, B=bright, m B=moderately bright, F=faint, v F=very faint, D=dark, v=very, m=moderately.

Two kinds of spots were observed in 1893, namely: (1) Dark spots upon a conspicuous double belt in the northern hemisphere; and (2) bright spots in the equatorial zone. It will be advisable to consider each of these separately.

The Dark Spots in the Northern Hemisphere.

These were somewhat numerous, and consisted of small dark spots upon a prominent double belt in the northern hemisphere. According to my observations the spots were nearly invariably double, or arranged in pairs, one on each of the two bands of the double belt, the component spots of each pair lying nearly north and south with respect to each other. On favourable nights the spots were seen double in this manner very distinctly, some of the plainer ones being in fact quite dark and definite. But with poor definition it was often difficult to see the spots distinctly double, the appearance being then that of one larger and more diffuse spot extending over the whole width of the double belt. Both M. Quénisset at Juvisy and M. Rudaux at Donville have, however, figured single spots situated upon only one band of the belt, the companion spots on the other band not having apparently been seen. One of the spots was also observed by Mr. A. Henderson at Liverpool, so that altogether they were seen by four different observers. The darkest and best defined of these spots were, in fact, comparatively easy to observe, and the accordance of the observations is, on the whole, very satis-

* With the exception of those cases where the interval between the two observations is very short, such as 2–6 days; since in such short intervals the unavoidable accidental errors of the observations have an enormously disproportionate effect upon the result.

factory. The Rev. A. Freeman also noted once that when the double N belt was best seen "its bands were mottled." A similar appearance seems to have been seen by some other observers.

Dark Spot α.

Date.	G.M.T. of Transit.	Long.	Weight.	Obs. reduced to May 16.	D'ff.	Period of Rotation.	
	h m	°		h m	m	h m s	
May 7	10 11·5	66·8	2	9 16·7	−0·2	10 14 27·1	(*a*)
13	9 32·0	72·5	3	13·8	−3·1	31·9	(*b*)
16	9 14·1	76·3	4	14·1	−2·8	33·8	(*c*)
June 8	10 29·0	109·9	½	9 23 0	+6·1	10 14 40·1	(*d*)

Description.—(*a*) v S, v F, v delicate. (*b*) 2 e S, F spots, v delicate. (*c*) 2 v S, F spots, well seen. (*d*) Glimpsed.

The mean period of rotation is $10^h\ 14^m\ 32{\cdot}1^s \pm 1^s{\cdot}22$, the number of rotations completed by the planet between the extreme observations being 75.

Dark Spot β.

Date.	G.M.T. of Transit	Long.	Weight.	Obs. reduced to April 14.	Diff.	Period of Rotation.	
	h m	°		h m	m	h m s	
Mar. 21	12 19·5	34·1	4	9 49·1	− 2·8	10 14 32·8	(*a*)
Apr. 5	(10 35)	(45·8)	...	(38·7)	(−13·2)	...	(*b*)
14	9 53·0	64·9	3	53·0	+ 1·1	31·5	(*c*)
May 4	11 15·0	89·6	1	54·4	+ 2·5	29·8	(*d*)
22	9 22·4	110·5	3	53·4	+ 1·5	30·7	(*e*)
25	9 0·1	111·7	3	9 49·5	− 2·4	10 14 27·8	(*f*)

Description.—(*a*) 2 e S, m D spots. (*b*) Obs. by Henderson. (*c*) 2 e S spots. South spot F, N. one v F; projecting S. of belt. (*d*) v S, F, not seen dup. (*e*) 2 e S, m D spots, seen v distinctly. (*f*) 2 e S, m D spots.

The simple mean of the rotation periods in col. 7 is $10^h\ 14^m\ 30^s{\cdot}5 \pm 0^s{\cdot}57$ (152 rotations). This spot, or actually pair of spots, was unusually definite, so that it could be well observed. The observations consequently agree very satisfactorily. In fact the accordance is really remarkable, as will be seen from the residuals in the 6th column. The observation of April 5 was made by Mr. Arthur Henderson at Liverpool with a $10\frac{1}{2}$-in. reflector, power 336. At $10^h\ 35^m$ "the N. dark belt appeared slightly darker on the meridian."*

* *English Mechanic*, April 28, 1893, p. 225.

Dark Spot γ.

Date.	G.M.T. of Transit.	Long.	Weight.
	h m	°	
Mar. 24	12 25·3	52·2	3 (*a*)
Apr. 20	9 51·4	93·0	4 (*b*)
26	9 4·7	94·6	1 (*c*)

Description.—(*a*) v S, m D, dup., plain. (*b*) 2 v S, m D spots. (*c*) Plain.

The observations are insufficient to give a reliable period of rotation. Comparing the first observation with the second and third, the resulting periods of rotation are respectively $10^h\ 14^m\ 44^s{\cdot}7$ and $10^h\ 14^m\ 34^s{\cdot}8$. Giving half-weight to the latter result, the period of the spot would be $10^h\ 14^m\ 41^s{\cdot}4$.

Dark Spot δ.

Date.	G.M.T. of Transit.	Long.	Weight.	Obs. reduced to May 10.	Diff.	Period of Rotation.	
	h m	°		h m	m	h m s	
Apr. 29	9 26·1	121·7	2	11 43·1	−0·7	10 14 33·4	(*a*)
May 8	8 35·0	135·0	1	47·5	+3·7	30·9	(*b*)
10	11 46·3	136·9	1	46·3	+2·5	30·1	(*c*)
13	11 20·3	136·0	1	38·7	−5·1	36·7	(*d*)
22	10 30·0	150·1	2	44·4	+0·6	30·1	(*e*)
25	10 10·3	152·9	3	11 43·0	−0·8	10 14 27·8	(*f*)

Description.—(*a*) S, F, susp. dup. (*b*) v S, m D, probably dup., quite plain. (*c*) Probably F, but def. bad. (*d*) v S, v F, v delicate. (*e*) v S, F, probably dup. (*f*) v S, F, probably dup.

Mean period of rotation$=10^h\ 14^m\ 30^s{\cdot}8\pm0^s{\cdot}84$ (61 rotations). The accordance of the observations is very satisfactory.

Dark Spot ε.

Date.	G.M.T. of Transit.	Long.	Weight.	Obs. reduced to May 5.	Diff.	Period of Rotation.	
	h m	°		h m	m	h m s	
Feb. 8	14 58·1	47·0	1	9 13·9	+2·3	10 14 26·8	(*a*)
May 5	9 11·9	142·2	3	11·9	+0·3	27·7	(*b*)
16	11 24·0	152·5	½	8·1	−3·5	29·3	(*c*)
June 9	8 59·7	182·2	1	9 12·7	+1·1	10 14 29·4	(*d*)

Description.—(*a*) Probably double. (*b*) v S (or S), m D, unusually plain. (*c*) S, F. (*d*) v S, v F, v delicate.

Giving to the observation of February 8 weight 3, because of the long interval separating it from the others, the mean period of rotation$=10^h\ 14^m\ 27^s{\cdot}7\pm0^s{\cdot}36$ (283 rotations). The observations, though not numerous, are again very accordant,

and as they extend over a long interval of time (121 days), the above period is a reliable one.

Dark Spot ζ.

Date.	G.M.T. of Transit.	Long.	Weight.	Obs. reduced to May 8.	Diff.	Period of Rotation.	
	h m	°		h m	m	h m s	
Apr. 23	11 17·5	158·0	3	9 61·1	+1·9	10 14 55·4	(*a*)
May 8	9 56 7	182·9	1	56 7	−2·5	63·8	(*b*)
14	9 31·5	196·9	2	61·9	+2·7	63·0	(*c*)
26	8 26·5	216·2	1	9 57·3	−1·9	10 14 54·5	(*d*)

Description.—(*a*) v S, F, not seen clearly double. (*b*) v S, F. (*c*) 2 v S, F spots. (*d*) v S, F?

Mean period of rotation = $10^h\ 14^m\ 58^s{\cdot}6 \pm 1^s{\cdot}56$ (77 rotations). Observations again very accordant.

Dark Spot η.

Date.	G.M.T. of Transit.	Long.	Weight.	Obs. reduced to Apr. 21.	Diff.	Period of Rotation.	
	h m	°		h m	m	h m s	
Mar. 31	10 13·0	128·6	4	8 29·0	−1·1	10 15 5·6	(*a*)
Apr. 21	8 32·0	171·2	½	32·0	+1·9	10 15 3·0	(*b*)
May 5	10 45·5	197·1	3	29·2	−0·9	10 15 1·2	(*c*)
17	(9 45·9)	(220·8)	...	(8 28·0)	(−2·1)	...	(*d*)

Description.—(*a*) 2 e S, m D spots, together quite plain. (*b*) No description, obs. v rough. (*c*) v S, F, well seen. (*d*) Obs. by Quénisset.

Mean period of rotation = $10^h\ 15^m\ 3{\cdot}7^s \pm 1^s{\cdot}01$ (82 rotations). The observations though few in number are very accordant. The last one is the result of measures of a sketch made by M. Quénisset at Juvisy with a 9·4-in. refractor,* and is in excellent agreement with my own observations. In this, as in all similar cases, the measures have been made and reduced prior to any comparison with other observations, so that there is no possibility of the resulting position having been affected by bias. M. Quénisset draws the spot on the northernmost of the two belts, and apparently did not notice the companion spot on the southernmost belt.

Dark Spot θ.

Date.	G.M.T. of Transit.	Long.	Weight.	
	h m	°		
Apr. 18	9 50·0	202·5	½	(*a*)
23	12 48 8	211·6	1	(*b*)
May 17	10 48·2	256·3	...	(*c*)

Description.—(*a*) S, D. (*b*) Apparently D, def. v confused. (*c*) Obs. by Quénisset.

* *L'Astronomie*, 1893, p. 327, fig. 124.

Observations insufficient to furnish a reliable period of rotation. Comparison of the two first observations makes it $10^h\ 14^m\ 56^s$, which agrees satisfactorily with the periods of the neighbouring spots. M. Quénisset's observation is based on measures of two sketches.* From one sketch the transit occurred at $10^h\ 55^m{\cdot}1$, and from the other at $10^h\ 41^m{\cdot}4$. The mean of these is the time given in the table, and it accords well with the other observations.

Dark Spot ι.

Date.	G.M.T. of Transit.	Long.	Weight.	Obs. reduced to May 11.	Diff.	Period of Rotation.	
	h m	°		h m	m	h m s	
Apr. 24	9 33·5	221·9	1	11 28·5	−7·1	10 14 60·1	(*a*)
27	9 31·1	235·0	3	42·0	+6·4	43·3	(*b*)
May 11	11 35·2	255·1	2	35·2	−0·4	54·8	(*c*)
14	11 14 2	257·2	1	30·0	−5·6	52·4	(*d*)
June 7	9 21·5	305·5	3	11 42·4	+6·8	10 14 60·5	(*e*)

Description.—(*a*) v S, F. (*b*) S, F, nearly certainly dup. (*c*) v S, m D? (*d*) Probably v S, F. (*e*) v S, m D, well defined, probably dup.

Mean period of rotation $=10^h\ 14^m\ 53^s{\cdot}35 \pm 2^s{\cdot}40$ (103 rotations). The observations are not so accordant as in the case of the previous spots.

Dark Spot κ.

Date.	G.M.T. of Transit.	Long.	Weight.	
	h m	°		
Apr. 30	9 31·4	249·6	½	(*a*)
May 3	9 23·7	259·5	3	(*b*)
9	8 39·0	262·2	½	(*c*)

Description.—(*a*) v S, F. (*b*) 2 e S, F spots. (*c*) v S? m D?

Observations insufficient to give a reliable period of rotation.

Dark Spot λ.

Date.	G.M.T. of Transit.	Long.	Weight.	Obs. reduced to Apr. 30.	Diff.	Period of Rotation.	
	h m	°		h m	m	h m s	
Apr. 18	11 38·5	266·2	1	10 34·1	+2·0	10 14 48·2	(*a*)
30	10 33·7	286·1	3	33·7	+1·6	41·5	(*b*)
May 6	9 53·8	291·6	2	25·9	−6·2	49·7	(*c*)
12	9 30·8	306·9	3	10 34·8	+2·7	10 14 65·9	(*d*)

Description.—(*a*) v S, F. (*b*) Dup. South spot v S, F; N. spot e S, F; projecting south of belt. (*c*) v S, F, probably dup. Time possibly 2^m in error. (*d*) 2 e S, F spots perhaps projecting south of belt.

* *L'Astronomie*, 1893, p. 327, figs. 124 and 125.

Mean period of rotation = $10^h\ 14^m\ 52^s{\cdot}2 \pm 3^s{\cdot}97$ (56 rotations). The observations are in good agreement. On two nights the spot projected south of the southernmost edge of the belt.

Dark Spot μ.

Date.	G.M.T. of Transit.	Long.	Weight.
	h m	°	
Apr. 27	11 6·0	290·6	1 (*a*)
May 18	9 16·7	327·4	1 (*b*)

Description.—(*a*) v S, F. (*b*) v S, v F, v delicate.

The period of rotation from these two observations is $10^h\ 14^m\ 55^s{\cdot}0$ (49 rotations), which agrees very closely with the periods of the neighbouring spots.

Dark Spot ν.

Date.	G.M.T. of Transit.	Long.	Weight.	Obs. reduced to May 3.	Diff.	Period of Rotation.	
	h m	°		h m	m	h m s	
Apr. 19	9 15·7	307·2	1	11 30·4	−0·7	10 14 63·4	(*a*)
30	11 44·4	327·6	2	29·3	−1·8	59·2	(*b*)
May 3	11 36·5	337·4	3	36·5	+5·3	62·1	(*c*)
27	9 29·5	17·9	1	11 28·4	−2·7	10 14 57·2	(*d*)

Description.—(*a*) v S, v F, v delicate. (*b*) v S, F, rather ill defined. (*c*) v S, v F, probably dup. (*d*) v S, m D, well defined.

Mean period of rotation = $10^h\ 15^m\ 0^s{\cdot}9 \pm 0^s{\cdot}81$ (85 rotations). The observations are in very satisfactory accordance.

Dark Spot o.

Date.	G.M.T. of Transit.	Long.	Weight.
	h m	°	
May 12	11 8·3	4·2	½ (*a*)
18	10 29·7	10·2	½ (*b*)

Description.—(*a*) v S, F. (*b*) Probably v S, glimpsed through cloud.

Observations insufficient to give the period of rotation with any exactness.

Dark Spot π.

Date.	G.M.T. of Transit.	Long.	Weight.	Obs. reduced to May 4.	Diff.	Period of Rotation.	
	h m	°		h m	m	h m s	
Mar. 29	12 11·4	308·4	2	8 40·9	−4·0	10 14 43·6	(*a*)
May 4	8 43 3	0·6	1	43·3	−1·6	40·7	(*b*)
10	8 18·3	14·8	½	53·1	+8·2	38·0	(*c*)
June 2	9 20·5	41·2	2	8 42·3	−2·6	10 14 36·2	(*d*)

Description.—(*a*) v S, F, dup. (*b*) S, D, dup? plain. (*c*) Probably F, v rough obs. (*d*) v S, m D, well defined, probably dup, plain.

Mean period of rotation=$10^h\ 14^m\ 40^s{\cdot}3 \pm 1^s{\cdot}27$ (152 rotations), the first observation having been given weight 3 on account of the long interval separating it from the others.

Dark Spot ρ.

Date.	G.M.T. of Transit.	Long.	Weight.
	h m	°	
May 1	9 34·0	16·0	3 (*a*)
June 8	9 17·0	67·6	4 (*b*)

Description.—(*a*) 2 e S, F spots, the N one slightly prec. the other. (*b*) 2 e S, m D, well defined spots, well seen.

Comparing these two observations together, the resulting period of rotation is $10^h\ 14^m\ 37^s{\cdot}8$ (89 rotations). As the observations both bear a high weight, and are separated by a considerable interval, this result is probably pretty exact.

Dark Spot σ.

Date.	G.M.T. of Transit.	Long.	Weight.	Obs. reduced to May 4.	Diff.	Period of Rotation.	
	h m	°		h m	m	h m s	
Feb. 7	15 20 3	295·1	2	9 47·4	− 7·8	10 14 33·8	(*a*)
May 4	9 54·5	42·4	3	54·5	− 0·7	31·8	(*b*)
10	9 28·3	55·9	3	65·2	+ 10·0	27·1	(*c*)
June 2	10 21·5	77·0	1	9 53·7	− 1·5	10 14 27·0	(*d*)

Description.—(*a*) 2 e S, v F spots. (*b*) 2 e S, v F spots. (*c*) v S, F, probably dup. (*d*) v S, v F, v difficult.

Mean period of rotation=$10^h\ 14^m\ 31^s{\cdot}1 \pm 1^s{\cdot}14$ (269 rotations), giving the first result weight 5 instead of 2 because of the long interval separating the observation from the others. The observations are not so accordant as usual, though, as they extend over a long interval of time, the period is a pretty reliable one.

Of the above spots, γ, θ, κ, μ, o and ρ may be dismissed from further consideration, since the observations of these spots are not sufficient to give the rotation periods with accuracy; although in every case, it should be observed, the observations of these spots accord excellently with those of neighbouring spots. There remain, then, eleven spots of which the periods of rotation are well ascertained, and these range themselves naturally into two classes or groups, according to the lengths of such periods, with one spot π nearly intermediate between the two groups. Thus we have:—

	Synonym.	Period of Rotation. h m s s	No of Rotations.
Class A.	σ	10 14 31·1 ± 1·14	269
	α	10 14 32·1 ± 1·22	75
	β	10 14 30·5 ± 0·57	152
	δ	10 14 30·8 ± 0·84	61
	ϵ	10 14 27·7 ± 0·36	283
Class B.	ζ	10 14 58·6 ± 1·56	77
	η	10 15 3·7 ± 1·01	82
	ι	10 14 53·35 ± 2·40	103
	λ	10 14 52·2 ± 3·97	56
	ν	10 15 0·9 ± 0·81	89
	π	10 14 40·3 ± 1·27	152

Combining the different results, the following are the final values for the two groups :—

		h m s s
Class A.	Mean Period of Rotation ...	= 10 14 29·07 ± 0·27
Class B.	„ „ „ ...	= 10 15 0·74 ± 0·56

there being rather more than half a minute (31^{s}·67) between the two.

The different results which compose each group agree closely *inter se*, the somewhat greater divergences in the case of Class B being due to the fact that the observations of the spots do not extend over such long intervals of time as do those of the spots forming Class A. It is clear, therefore, that the surface material rotated more than half a minute more quickly in the same latitude upon one side of *Saturn* than upon the other. Similar differences are common enough upon *Jupiter*, though even with this planet it is rare to find the difference so considerable and so well marked as in the present instance. Taking the date May 5 as a convenient epoch, the spots of Class A extend from longitude 45° to 140°, and those of Class B from longitude 175° to 340°, the intermediate region, exemplified by spot π, extending from about longitude 340° to 45°.

It will be well to explain here why there are so few observations in March and the early part of April, and why, in the case of closely adjacent spots, there are no instances of both spots having been observed upon the same night. The paucity of observations previous to the middle of April was due to poor definition, although otherwise the weather was remarkably favourable to observation, cloudy nights being, in fact, quite exceptional. The other peculiarity resulted from excess of caution. Not infrequently, after one spot had been observed, another one was seen close to it. But in such cases it was feared that the appearances might be deceptive, and the observations were either discontinued for an hour or two, in order to give

the eye a complete rest, or else confined for a time to the bright equatorial spots.

The Bright Equatorial Spots.

These were spots slightly brighter than the average surface of the bright equatorial zone, and resembling in appearance the bright spots which form such a characteristic feature of the corresponding region of *Jupiter*. Roughly circular in shape, they averaged about 2″ in diameter—perhaps a little less—but as they were usually very indefinite on the edges, it was generally difficult to distinguish a distinct boundary. Occasionally they were recorded as being very white, but more usually they appeared slightly yellowish in tint. Like the spots upon *Jupiter*, they seem to be variable in brightness.

Satisfactory observations of these spots were made in 1893 by four observers in addition to the writer, namely, Mr. H. MacEwen at Glasgow, Mr. G. L. Brown at Stirling, the Rev. P. H. Kempthorne at Wellington College, and M. Quénisset at Juvisy. Three or four other observers also saw the spots, but have not recorded times or positions.* Generally speaking, these bright spots were more indefinite than the dark spots previously treated of, and consequently the observations are not quite so accordant.

Bright Spot No. 1.

Date.	G.M.T. of Transit.	Long.	Weight.	Obs. reduced to Mar. 31.	Diff.	Period of Rotation.	
	h m	°		h m	m	h m s	
Feb. 7	12 49·3	206·6	2	10 66·6	+ 7·6	10 12 56·6	(*a*)
Mar. 22	12 27·9	163·9	2	59·6	+ 0·6	57·7	(*b*)
31	10 59·5	155·9	1	59·5	+ 0·5	57·4	(*c*)
Apr. 23	10 30·1	130·2	1	50·8	− 8·2	61·4	(*d*)
29	9 28·8	123·3	3	48·1	−10·9	64·6	(*e*)
May 4	12 14·8	124·7	1	58·4	− 0·6	59·5	(*f*)
5	8 45·8	126·9	3	63·4	+ 4·4	55·0	(*g*)
13	10 50·2	118·3	3	61·1	+ 2·1	60·6	(*h*)
16	10 18·4	114·0	3	58·4	− 0·6	59·5	(*i*)
	(10 29·3)	(120·4)	...	(69·3)	(+10·3)	...	(*j*)
22	9 20·8	109·5	2	59·8	+ 0·8	61·0	(*k*)
25	8 44·8	102·7	1	52·7	− 6·3	56·3	(*l*)
June 8	10 9·0	98·1	½	66·4	+ 7·4	66·8	(*m*)
11	9 25·7	86·9	½	10 51·8	− 7·2	10 12 55·4	

Description.—(*a*) S, m B?, plain. (*b*) Probably m B. (*c*) v S, F. (*d*) No description, def. confused. (*e*) S, F, perhaps a double nucleated spot. (*f*) S, F. (*g*) S, m B, rather well defined, plain. (*h*) S, F, plain. (*i*) S, F, v white. (*j*) Obs. by Kempthorne. (*k*) v S, F. (*l*) S, m B, well defined, rich white colour, plain. (*m*) S, v F ?, v. rough obs.

* In 1892 the Rev. A. Freeman secured observations of several of these

Mean period of rotation = $10^h\ 12^m\ 59^s{\cdot}4 \pm 0^s{\cdot}61$ (291 rotations). The spot clearly did not move at a perfectly uniform rate. Thus between February 7 and April 29 its motion was a little swifter than that corresponding to the above period. Between the latter date and May 5 there occurred a rather sudden shift or displacement to the eastward, causing a difference of about 15^m in the time of the transit over the central meridian of the disc. Subsequent to May 5 the motion of rotation conformed closely to the adopted period of rotation of $10^h\ 12^m\ 59^s{\cdot}4$. The second observation of May 16 is the result of measures of a sketch made by the Rev. P. H. Kempthorne at Wellington College, with an 8½-inch reflector—power, 350. The last observation is derived from measures of a sketch published in *l'Astronomie*, 1893, p. 329, fig. 126.*

Bright Spot No. 2.

Date.	G.M.T. of Transit.	Long.	Weight.	Obs. reduced to May 4.	Diff.	Period of Rotation.	
	h m	°		h m	m	h m s	
Mar. 10	13 8·0	128·8	2	10 63·1	+1·9	10 12 60·5	(*a*)
30	13 19·9	113·4	3	65·5	+4·3	60·4	(*b*)
Apr. 8	11 45	101·5	1	58·0	−3·2	63·4	(*c*)
9	8 8·9	99·6	½	55·9	−5·3	64·6	(*d*)
May 1	...	...	...	...	...	...	(*e*)
4	11 9·1	86·2	1	69·1	+7·9	59·1	(*f*)
7	10 24·8	74·6	3	53·8	−7·4	65·0	(*g*)
13	9 35·5	74·5	2	62·0	+0·8	59·0	(*h*)
16	9 7·1	72·2	4	62 4	+1·2	62 9	(*i*)
27	10 42·0	60·4	1	58·1	−3·1	58·4	(*j*)
June 2	9 50·5	58·8	1	10 63·9	+2·7	10 12 63·8	(*k*)

Description.—(*a*) v S, F or perhaps m B. (*b*) S, m B, seen brightly. (*c*) Obs. by MacEwen. (*d*) Obs. by Brown. (*e*) Seen, transit not obs. (*f*) S, m B, seen brightly. (*g*) S, F, v white. (*h*) S, F. (*i*) S, m B, well defined, quite plain and brilliant. (*j*) Probably v S, F, def. confused. (*k*) v S, v F, v difficult, definition poor.

Mean period of rotation = $10^h\ 13^m\ 1^s{\cdot}8 \pm 0^s{\cdot}49$ (197 rotations). The observation of April 8 was made by Mr. H. MacEwen at Glasgow, with a 5-inch refractor. The spot was observed in

bright equatorial spots. One of them was "easily seen" once by Professor Porro with the 11-in. refractor of the Turin Observatory. Mr. G. T. Davis, of Reading, even suspected one with only a 3¾-in. refractor. The Rev. W. R. Waugh frequently saw them at Portland with a 12½-in. reflector, both in 1892 and 1893.

* This sketch was made by M. Rudaux at Donville, but by a mistake it was attributed to Mr. Arthur Mee, and it is not clear at what time it was actually made. As, however, the calculations had been completed, and the adopted rotation period appears to satisfactorily represent all the other observations, no alteration has been made on this account.

mid-transit at the time given. The observation of April 9 was made by Mr. G. L. Brown at Stirling, with a $10\frac{1}{4}$-inch reflector. At 9^h the spot was seen about 30° to the west of the central meridian. This spot was therefore observed by three different observers.

Bright Spot No. 3.

Date.	G.M.T. of Transit.	Long.	Weight.	Obs. reduced to April 20.	Diff.	Period of Rotation.	
	h m	°		h m	m	h m s	
Mar. 16	12 33·4	137·9	2	10 13·1	−2·8	10 12 60·7	(*a*)
19	12 9·4	138·4	3	18·7	+2·8	56·1	(*b*)
31	10 9·1	126·3	4	16·8	+0·9	58·2	(*c*)
Apr. 20	10 15·7	107·2	2	15·7	−0·2	57·8	(*d*)
May 22	8 25·3	77·0	1	14·2	−1·7	58·3	(*e*)
June 8	9 8·7	62·8	3	10 16·7	+0·8	10 12 59·3	(*f*)

Description.—(*a*) v S, m B, well defined. (*b*) v S, m B. (*c*) S, m B, quite bright and easy. (*d*) v S, F, difficult. (*e*) S, m B, v white. (*f*) S, F, v white, plain.

Mean period of rotation$=10^h\ 12^m\ 58^s{\cdot}3\pm0^s{\cdot}42$ (197 rotations). The observations are remarkably accordant.

Bright Spot No. 4.

Date.	G.M.T. of Transit.	Long.	Weight.	Obs. reduced to March 29.	Diff.	Period of Rotation.	
	h m	°		h m	m	h m s	
Feb. 19	14 29·0	323·5	1	11 19·3	−0·9	10 12 46·5	(*a*)
Mar. 20	12 54·4	289·7	3	21·2	+1·0	45·4	(*b*)
29	11 20·3	278·4	1	20·3	+0·1	45·2	(*c*)
Apr. 27	...	...	...	...	...	...	(*d*)
May 14	10 14·5	222·2	4	19·2	−1·0	45·9	(*e*)
17	(9 40·8)	(216·8)	...	(16·1)	(−4·1)	...	(*f*)
June 9	9 16·3	192·0	1	11 21·1	+0·9	10 12 46·5	(*g*)

Description.—(*a*) S, F, seen plainly. (*b*) S, F, well seen. (*c*) S, F, seen plainly. (*d*) Suspected. (*e*) S, m B, plain. (*f*) Obs. by Quénisset. (*g*) v S, v F, seen quite distinctly.

Mean period of rotation$=10^h\ 12^m\ 45^s{\cdot}8\pm0^s{\cdot}14$ (258 rotations). The accordance of the observations is very remarkable, none of the six observations made by the writer differing by more than one minute from the mean. The time of May 17 is derived from measures of a sketch made by M. Quénisset with the 9·4-inch refractor of the Juvisy Observatory.*

* *L'Astronomie*, 1893, p. 327, fig. 124.

Bright Spot No. 5.

Date.	G.M.T. of Transit.	Long.	Weight.	Obs. reduced to Apr. 8.	Diff.	Period of Rotation.	
	h m	°		h m	m	h m s	
Mar. 4	13 27·5	111·0	1	10 61·9	+7·3	10 12 47·7	(*a*)
10	12 15·5	98·0	2	49·9	−4·7	57·9	(*b*)
18	14 19·0	89·6	1	50·7	−3·9	53·5	(*c*)
Apr. 8	10 51·1	69·9	1	51·1	−3·5	55·9	(*d*)
	11 0·9	75·6		60·9	+6·3		(*e*)
20	8 58·7	62 1	1	58·0	+3·4	54 8	(*f*)
May 10	8 52·0	34·6	3	45·2	−9·4	52·6	(*g*)
27	9 43·0	25·8	2	10 59·2	+4·6	10 12 59·9	(*h*)

Description.—(*a*) S, F, seen clearly but def. unsteady. (*b*) F, perhaps brighter. (*c*) S, F, seen plainly. (*d*) Obs. by Brown. (*e*) Obs. by MacEwen. (*f*) S, m B, seen brightly and plainly. (*g*) S, m B, seen brightly and plainly. (*h*) S, F, rather well defined.

Mean period of rotation $=10^h\ 12^m\ 55^s{\cdot}0 \pm 0^s{\cdot}98$ (197 rotations). The first observation of April 8 was made by Mr. G. L. Brown at Stirling, with a $10\frac{1}{4}$-inch reflector. At 10^h the spot was seen about 30° east of the central meridian. The second observation is derived from measures of a sketch made by Mr. H. MacEwen at Glasgow, with a 5-inch refractor. As the two positions agree pretty well with each other, the mean of the two has been adopted in the calculations with weight 1.

Arranging the different bright equatorial spots in order of increasing longitudes, we have the following results :—

	No.	Period of Rotation.	No. of Rotations.
		h m s s	
Class C	5	10 12 55·0 ± 0·97	197
	2	10 13 1·8 ± 0·49	197
	3	10 12 58·3 ± 0·42	197
	1	10 12 59·4 ± 0·61	291
	4	10 12 45 8 ± 0·14	258

The first four spots evidently form a group, and combining the different values, the concluded period of rotation for Class C is $10^h\ 12^m\ 59^s{\cdot}36 \pm 0^s{\cdot}27$. The period of the last spot is more than 13^s shorter than this, so that we have here a similar difference to that existing between groups A and B of the dark northern spots, only here the difference is not so great. Spot No. 4 appears to be fairly representative of a somewhat extensive region. Hence, taking the date May 5 as the epoch, the surface material in the equatorial zone of *Saturn* rotated at the rate of $10^h\ 12^m\ 59^s{\cdot}36 \pm 0^s{\cdot}27$ between longitudes 0° and 140°;

whilst between longitudes 140° and 360° it rotated slightly more quickly, the average period of rotation here being represented by that of spot No. 4, namely, $10^h\ 12^m\ 45^s{\cdot}8$.

If we compare these figures with those indicated by the observations of 1892 and 1891, there has evidently been a continual acceleration in the motion of the bright equatorial spots, corresponding to a decrease in the period of rotation. Thus, adopting Mr. Marth's figures for the latter years, we have—

	h	m	s
Period of rotation in 1891 =	10	14	21·8
„ „ „ „ 1892 =	10	13	38·4
„ „ „ „ 1893 =	10	12	59·4

Similar changes are not uncommon in the case of the planet *Jupiter*, though they rarely occur quite so rapidly. For instance, in the case of one such change in the corresponding region of *Jupiter*, a period of eight years was required for an increase of half a minute in the period of rotation.

It is not clear at present in what manner these changes occur upon *Saturn*. They might take place—(1) by a steady or continual increase or decrease; (2) by sudden changes in the rate of motion analogous to those lately well observed in the case of *Jupiter*; * (3) by the old spots dying out and new ones successively making their appearance endowed with a swifter or slower motion. The first of these three ways may be dismissed at once, the observations showing that there has been no such continual change. So that the question would seem to come under one of the two latter.

Summary.

The principal conclusions derived from the foregoing discussion may be summarised as follows:—

(1) Between N. Kronocentric latitudes 17° and 37° the surface material of *Saturn* rotated in 1893 at the rate of $10^h\ 14^m\ 29^s{\cdot}07 \pm 0^s{\cdot}27$ between longitudes 45° and 140°, and at the rate of $10^h\ 15^m\ 0^s{\cdot}74 \pm 0^s{\cdot}56$ between longitudes 175° and 340°, whilst between longitudes 340° and 45° there was a region in which the surface material rotated at a rate intermediate between the above values.

(2) Between N. Kronocentric latitude 6° and about 2° S. latitude the surface material of *Saturn* rotated in 1893 at the rate of $10^h\ 12^m\ 59^s{\cdot}36 \pm 0^s{\cdot}27$ between longitudes 0° and 140°, whilst between longitudes 140° and 360° the rate of rotation was rather quicker, the average period of rotation here being well represented by that of spot No. 4, namely, $10^h\ 12^m\ 45^s{\cdot}8$.

* See *Observatory*, 1892, p. 110. In the case of one spot a change of over half a minute in the period of rotation occurred quite suddenly, and the region affected by the sudden change extended over more than 90° of zenographical longitude.

The above longitudes are referred to May 5 as a convenient epoch. The limiting Kronocentric latitudes have been kindly calculated by the Rev. A. Freeman from measures of two drawings of *Saturn* by means of the tables published by him in the *Monthly Notices*, vol. liv. p. 30.

Results of Micrometer Measures of Double Stars with the $12\frac{3}{4}$*-inch Merz Refractor, at the Royal Observatory, Greenwich, during the year* 1893.

(*Communicated by the Astronomer Royal.*)

Σ [OΣ]	Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Magnitude.		Position Angle. °	Distance. ″	No. of nights.	Fraction of year 1893.
180	γ Arietis	1 48·0	71 12	4·2	4·4	358·1	6·83	2	·097
305		2 41·7	60 10	7·3	8·2	317·1	2·83	1	·126
[82]		4 17·0	75 17	7·0	9 0	149·5	1·23	1	·170
535		4 18·0	78 52	6·7	8·2	324·7	1·17	2	·175
572		4 32 3	63 21	6·5	6·5	201·9	3·23	2	·128
577		4 35·4	52 41	7·7	7 7	71·6	1·64	2	·140
[92]		4 53·3	50 45	6·0	9 7	253·4	2·01	2	·184
[98]		5 2·4	81 39	6 0	6 8	188 6	1·05	4	·197
742		5 30·5	68 5	7·5	8·0	258 7	3·43	2	·205
749		5 30·9	63 9	7·1	7·2	167·4	1·14	2	·154
932		6 28·6	75 10	8·2	8 3	328·1	2·34	3	·184
[149]		6 30·3	62 38	6 5	9·0	284·3	0 88	2	·189
945		6 33·3	48 56	7·1	8·0	269·0	0·54	2	·240
982	38 Geminorum	6 49·0	76 42	5·4	7·7	160·7	6·77	2	·216
[165]	45 Geminorum	7 2·6	73 55	5 0	10·7	48 6	3·62	1	·285
1037		7 6·6	62 35	7·1	7·1	308·7	1·30	4	·159
[170]		7 12·2	80 31	7·5	7·5	112·4	0·74	3	·153
1066	δ Geminorum	7 14·2	67 50	3·2	8·2	206 8	7·37	2	·231
1074		7 15·4	89 25	7·8	8·2	144·4	0·75	2	·216
1081		7 18·1	68 21	7·5	8·5	226·4	1 69	2	·205
1104		7 24·9	104 47	6 5	8·0	328·3	2·23	2	·229
1110	Castor	7 28·2	57 54	2 7	3 7	227·6	...	1	·101 *
						229·6	5·67	4	·186 *
[175]		7 28·6	58 51	6·0	6 6	329·9	0·50	3	·249
[176]		7 33·3	89 15	7·3	9·3	209·8	1·43	2	·241
[177]		7 34·7	52 21	7·5	8·5	124·3	0·51	3	·241
1126	Piazzi VII. 170	7 34·8	84 33	7 2	7·5	142·5	1·12	4	·206

Σ [OΣ]	Name.	R.A. 1900.	N.P.D. 1900.	Magnitude.		Position Angle.	Distance.	No. of nights.	Fraction of year 1893.	
		h m	° ′			°	″			
[182]		7 47·4	71 45	7·0	7·5	31·9	1·26	2	·194	
1157		7 49·8	92 31	8·0	8·0	...	0·89	1	·224	*
						257·3	0·88	2	·229	
[185]		7 52 2	88 35	6·8	7·0	17·3	0·36	2	·241	
1175		7 57·3	85 33	7·7	8 8	221·9	1·47	2	·228	
[187]		7 57·7	56 41	6·9	7·5	285·2	0·34	3	·238	
	β 581 A.B.	7 58 8	77 25	8·5	8·6	273·1	0·84	1	·309	
1187		8 3·2	57 27	7·1	8·0	45·8	2 17	4	·141	
1196	ζ Canc. A.B.	8 6·5	72 3	5·0	5·7	26 5	1·05	2	·208	
	,, A.C.	...	...	5 0	5·5	116·0	5·23	2	·208	
	,, B.C.	...	...	5·7	5·5	131·3	5·17	2	·208	
1202	Piazzi VIII. 13	8 8·1	78 50	7·7	9 8	321·1	2·18	2	·211	
1211		8 11·1	50 52	9·1	9·6	309·7	0 74	2	·216	
1216		8 16·3	91 16	7·5	8·2	183·6	0·35	3	·245	
1223	ϕ^2 Canc.	8 20·7	62 44	6·0	6·5	217·0	4·35	2	·211	
1224	ν^1 Canc.	8 20·7	65 8	6·0	7·1	41·7	6·13	3	·243	
1228		8 21·6	62 5	8·3	8·9	350·4	9·27	2	·235	
1243		8 28·9	88 2	8·0	9·7	225·5	1·53	2	·246	
1263		8 38·5	47 55	7·6	8·2	20 2	49·91	1	·268	
1273	ε Hydræ	8 40·5	83 13	3·8	7·8	224·7	3·28	4	·240	
1291	57 Canc.	8 48 1	59 3	5·9	6·4	327·9	1·23	3	·244	
1300		8 54·7	74 20	8 7	8 8	200·1	5·17	2	·234	
3121		9 11·8	60 58	7·2	7·5	177·3	0·32	3	·265	
1333		9 12·2	54 13	6·6	6·9	226 0	1·68	2	·234	
1334	38 Lyncis	9 12·6	52 46	4·0	6·7	238·3	3·75	2	·244	
1338		9 14·6	51 23	7·0	7·2	165·7	1·52	2	·277	
[201]		9 17·9	61 33	7·5	9·0	226·5	...	1	·271	*
						219 5	1·50	2	·270	
1348		9 19·3	83 13	7·5	7·6	324·5	1·40	3	·260	
1355		9 22·1	83 19	7·2	7·2	333·8	2·30	3	·287	
1356	ω Leonis	9 23·1	80 30	6 2	7·0	109·2	0·50	2	·274	
1374		9 35·1	50 35	6·8	8·0	283·4	3·62	3	·300	
1389		9 46 7	62 32	8·2	9·0	317·1	2·15	2	·304	
1406		9 59·8	58 26	8·0	8·7	225·7	1·19	2	·303	
[215]	Piazzi X. 23	10 10·5	71 45	7·0	7·2	209·5	0·95	2	·268	
1424	γ Leonis	10 14·5	69 36	2 0	3 5	115·0	3·73	4	·301	
1429		10 19·5	64 52	8·2	8·2	250·6	1·19	2	·268	
[218]		10 22·1	85 54	7·3	9 2	90	0·3	1	·293	

Σ [0Σ]	Name.	R.A. 1900.	N.P.D. 1900.	Magnitude.	Position Angle.	Distance.	No. of nights.	Fraction of year 1893.
		h m	° ′		°	″		
1439		10 24·6	68 41	8·0 8·5	118·4	1·62	3	·305
1450	49 Leonis	10 29·8	80 49	6·0 8·7	157·8	2·41	4	·324
1457		10 33 6	83 45	7·6 8 5	316·3	0·96	3	·342
[224]	Piazzi X. 128	10 34·5	80 39	7·2 9·2	306 0	0·61	2	·305
[228]		10 41·9	66 54	7·2 8·1	198 6	0·77	2	·385
[229]		10 42·2	48 21	6 7 7 1	330·4	0·67	2	·335
1476		10 44·2	93 28	7·2 8·0	6 6	1·98	1	·353
1504	Piazzi X. 229	10 59·0	85 50	7·5 7·6	290·4	1 10	2	·349
1517	Piazzi XI. 9	11 8·6	69 20	7·3 7·0	275·1	0·68	1	·361
1523	ξ Ursæ Maj.	11 12·8	57 54	4·0 4·9	187·7	1·76	4	·345
1527		11 13·8	75 11	7·0 8·0	18·5	3·73	3	·411
1534		11 16 7	71 16	8·0 11·2	...	5 36	1	·370 *
					324·7	5·61	1	·370 *
1536	*i* Leonis	11 18·7	78 55	3·9 7·1	59·6	2·55	2	·383
1555	Piazzi XI. 111	11 31·0	61 40	6·4 6·8	356·8	0·48	3	·347
[237]		11 33·7	48 18	7·4 9·0	266·4	1·21	3	·416
1606		12 5·8	49 33	6·3 7·0	335·7	1·07	3	·361
[250]		12 20·1	46 22	7·7 8·0	331·4	0·42	5	·376
1643		12 22·3	62 25	8·0 8·3	41·7	2·01	2	·357
1647		12 25·6	79 44	7·5 7·8	219·6	1·32	2	·363
1658	A.B.	12 30·1	82 0	8·0 9 8	358·3	2·98	2	·335
1658	A.C.	12 30·1	82 0	...	280·1	0·27	2	·335
1663		12 32·3	68 15	7·8 8·7	112·4	0·77	2	·358
1670	γ Virginis	12 36·7	90 54	3·0 3·0	151·7	5·86	2	·319
1687	35 Comæ	12 48·4	68 12	5·0 7·8	73·1	1·49	3	·371
[256]		12 51·4	90 25	7·2 7·6	267·5	0·39	2	·418
1728	42 Comæ	13 5·1	71 57	6·0 6·0	199·5	0·47	2	·409
[261]		13 7·5	57 24	6·9 7·4	344·2	1·42	3	·419
1734		13 15·6	86 33	7·2 7·9	195 0	1·15	2	·460
1742		13 19·3	88 4	7·4 7·9	349·4	1·42	1	·463
[266]		13 23·5	73 46	7·3 7·8	335·3	1·54	2	·387
1757	Piazzi XIII. 127	13 29·2	89 48	7·8 8·9	80·0	2·63	2	·415
1777	84 Virginis	13 38·0	85 57	5 8 8·2	233·8	3·18	2	·427
1781		13 41·2	84 23	7·8 8·2	278·7	1·32	3	·462
	β 612	13 34·8	78 42	6·4 6 5	193·1	0·39	1	·468
1808		14 5·3	62 56	8·2 9·0	71·9	3·71	1	·463
1819		14 10·3	86 26	7·9 8 0	4·7	1·12	1	·468
1834		14 16·6	41 1	7·1 7 2	145·5	0·39	1	·468

Σ [OΣ]	Name.	R.A. 1900.	N.P.D. 1900.	Magnitude.	Position Angle.	Distance.	No. of nights.	Fraction of year 1893.
		h m	° ′		°	″		
1865	ζ Bootis	14 36·4	75 31	3·5 3·9	105·4	0·52	3	·488
1867		14 36·7	58 17	7·7 8·2	15·2	1·43	3	·455
1877	ϵ Bootis	14 40·6	62 30	3·0 6·3	328·8	2·63	2	·488
1888	ξ Bootis	14 46·7	70 29	4·7 6·6	235·5	3·70	4	·457
	β 31	14 7·8	70 52	8·4 9·7	196·3	1·87	4	·464
[287]		14 47·9	44 40	7·5 7·6	321·8	0 94	1	·493
[288]		14 48·5	73 53	6·4 7·1	191·3	1·68	2	·470
1932		15 14·1	87 52	5·6 6·1	137·3	0·92	2	·497
1937	η Coronæ	15 19·0	59 21	5·2 5·7	245·3	0·85	3	·497
1938	Piazzi XV. 74	15 20·8	52 18	6·7 7·3	82·9	1·26	1	·501
[296]		15 23·1	45 40	7·0 8·6	310·5	1·55	2	·479
1954	δ Serpentis	15 30·0	79 9	3·2 4·3	187·3	4·12	1	·496
[298]		15 32·5	49 52	7·0 7·3	170·6	1·03	1	·457
[303]		15 56·1	76 26	7·4 7 9	137·5	...	1	·466 *
					135·2	0·56	1	·466 *
2140	α Herculis	17 10·1	75 30	3·0 6·1	121·0	4·87	1	·731†
2203		17 38·3	48 17	7·5 7·8	328·2	0·93	1	·690
2215		17 41·1	72 15	5·9 7·9	164·1	0·39	1	·690
2272	70 Ophiuchi	18 0·4	87 28	4·1 6·1	308 9	2·34	1	·690

The measures were made with a position-micrometer on the 12¾-inch Merz refractor, which is mounted on the Lassell equatorial. In general the power of the eye-piece used was 412.

The measures were made, except where specially noted, by Mr. Lewis.

Measures marked * were made by Mr. Turner, those marked † by Mr. Edney.

The approximate positions are for the year 1900, and the magnitudes have been taken from Struve and Otto Struve.

Royal Observatory, Greenwich :
1894 *March* 9.

New Orbit of 9 Argûs, β 101. By Professor S. Glasenapp.

(*Communicated by T. Lewis.*)

In the article "Orbit of 9 *Argûs*, β 101," published in vol. lii. No. 8 of the *Monthly Notices of the Royal Astronomical Society*, I have communicated the elements of this close double star. These elements present a fairly good agreement with the observations, especially for the angles of position, but in the distances the last observation (0″·22) gives a discordance of 0″·16 with the calculated distance, which is 0″·38.

This circumstance was mentioned by Mr. S. W. Burnham, M.A., in an article published in the journal *Astronomy and Astrophysics*, No. 116; where, supposing that the position of the satellite for the year 1875 must be changed as follows—

instead of	1875·71 $\theta = 289°·7$ $\rho = 0''·46$
it must be	1875·27 $\theta = 289°·7$ $\rho = 0''·58$

he draws another ellipse with a larger eccentricity. I admit that these corrections made by Mr. Burnham are very probable. If we adopt them, and take his apparent ellipse, we obtain a new set of elements; if, then, we determine their corrections by the method of least squares, we obtain the following most probable elements of 9 *Argûs*:—

T = 1892·27	$\lambda = 73°·70$
P = 23·33 years	$i = 77°·73$
$n = +15°·430$	$e = 0·668$ ($\phi = 41°·943$)
$\Omega = 97°·54$	$a = 0''·60$

I give in the next table the comparison of these elements with the observations (see the above-mentioned note in the M.N.):—

t	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$
	°	°	°	″	″	″
1875·24	289·7	290·0	−0·3	0·58	0·58	0·00
78·50	302·2	300·0	+2·2	0·45	0·48	−0·03
79·68	306·2	305·1	+1·1	0·38	0·42	−0·04
82·21	319·7	323·4	−3·7	0·35	0·29	+0·06
83·11	336·2	334·6	+1·6	0·30	0·24	+0·06
89·08	76·4	76·0	+0·4	0·34	0·32	+0·02
90·22	83·8	84·7	−0·9	0·34	0·35	−0·01
91·06	91·5	90·6	+0·9	0·34	0·34	0
92·05	98·7	100·1	−1·4	0·25	0·25	− 003

The sum of the squares of the residuals is largely reduced; indeed, for our first set of elements it was 52·31 for θ and 0·0342 for ρ. For the new elements we have 26·13 for θ and 0·0111 for ρ.

In both angle and distance the sum of the squares has diminished very considerably. Therefore the new system of elements must be considered as representing much better the true orbit of 9 *Argûs*.

With these new elements the following positions of the satellite for several years have been calculated:—

	θ	ρ		t	θ	ρ
	°	″			°	″
1894·0	273·8	0·32		97·0	285·9	0·57
95·0	279·6	0·46		98·0	288·5	0·58
96·0	283·1	0·54		99·0	291·1	0·57

The angular velocity is increasing, and between 1892 and 1894 the angle of position has changed 180°; therefore each observation obtained during the present and next years will be of great value for the investigation of the orbit; but 9 *Argûs* belongs to the most difficult double stars, and is accessible only to the most powerful telescopes placed in perfect conditions.

St. Petersburg,
Observatory of the Imperial University:
1894 *March* 4.

On the Proper Motion of Stars in the Dumbbell Nebula. By Professor Arthur A. Rambaut, M.A., Sc.D., and W. E. Wilson, F.R.A.S.

Most photographs of the Dumbbell Nebula in *Vulpecula* (M. 27, N.G.C. 6853) represent the brighter portion of it as being of a sort of hour-glass form or approximately the figure which would be formed by two opposite sectors of a circle of about 60° each. The stars, too, which are found in the nebula are not distributed wholly without relation to the form of the latter, for it is a remarkable fact that exactly at the common centre of the two sectors, or the neck of the hour-glass, a comparatively conspicuous star is placed, while the extremities of each of the bounding arcs are approximately indicated by stellar points. These stars are respectively *d*, *a*, *m*, *o*, and *c* in the figure.

In the *Philosophical Transactions of the Royal Society*, Vol. 151, Part III., the Earl of Rosse publishes a note on the positions of twenty-six stars as determined at the Poulkova

Observatory by M. Otto Struve, in which the latter expresses his conviction that the probable error in the coordinates will be considerably less than one second of arc. As some of these stars lie outside the region over which the nebula extends on the photographs or that which it occupies on the Parsonstown drawing it appears an interesting inquiry whether those stars apparently related to the nebula indicate any proper motion, as compared with those lying outside it, in the forty-two years which have elapsed since M. Struve's measures were made.

In order to investigate this question we measured two photographs taken with the 2-foot equatorial reflector belonging to Mr. Wilson on 1893 September 3 and 8, with exposures of 27^m and 2^h respectively.

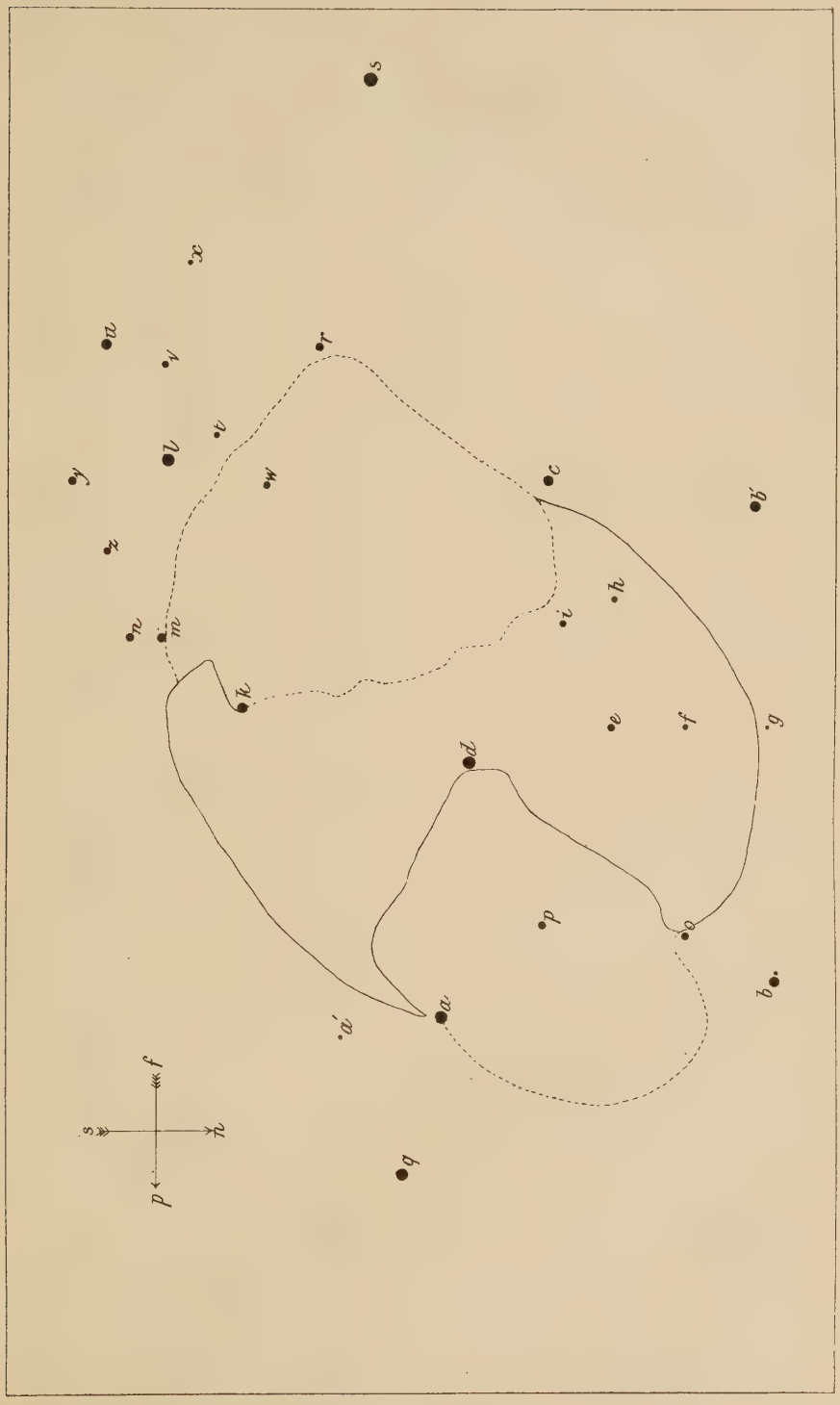
The plates were examined in the measuring microscope of the Dunsink Observatory, in which the coordinates are determined on two screws at right angles to each other. The plates were placed in position in the microscope by means of the relative positions determined by M. Struve for the stars *a* and *s* of his list, so that each screw was parallel to one of the axes of coordinates employed by that astronomer. The scale value of the photograph, too, was determined from the value of $\Delta\alpha \cos\delta$ given by him for the same stars, which, depending as it does on a larger number of observations than the other measures, is the best determined interval in the group, besides being very nearly the longest. Consequently any error affecting its length will be felt to a smaller extent in the measures of any other interval.

Our coordinates are thus directly comparable with those of M. Struve, our axis on the plate being, by the above process, set in the direction of the parallel passing through *a* at the date of his observations. The correction for refraction has not been applied, as it was found that in no case would its effect amount to a tenth of a second of arc, and the measures, though probably correct to within two or three tenths of a second, could hardly be relied on to a higher degree of accuracy.

A comparison of our results with those obtained by Mr. Bindon Stoney, using the 6-foot reflector belonging to the Earl of Rosse, and with those of M. Struve, is given in Table II. The former cannot be depended on with any high degree of precision on account of a difficulty in determining the parallel arising from the altazimuth form of the great reflector. They are, however, given in the table, as in several cases they point to the existence of a proper motion in the same direction as is deduced from a comparison of our measures with those of M. Struve.

It will be seen from the columns headed $X_s - X_w$ and $Y_s - Y_w$ that in most cases there are discrepancies which can hardly be attributed to errors of observation. These would indicate in the case of the stars *p*, *d*, *k*, *i*, *b'*, and *y* proper motions amounting in forty-two years to $4''{\cdot}6$, $3''{\cdot}9$, $5''{\cdot}7$, $6''{\cdot}0$, $5''{\cdot}2$, and $5''{\cdot}3$ respectively.

It does not seem, however, that there is any obvious relation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between the position which a star bears with regard to the nebula and the amount or direction of its proper motion, but further observations will be necessary to decide this point.

In the accompanying plate the positions of the stars are plotted down from our measures, and the form of the nebula is indicated in outline to facilitate the identification of the stars.

TABLE I.—*Coordinates of Stars in the Dumbbell Nebula.*

Name.	X	Y	Name.	X	Y
q	− 94″·4	− 20″·6	*h*	+ 247″·1	+ 102″·6
	94·4	20·6		246·7	104·4
	− 94·4	− 20·6		+ 246·9	+ 103·5
a'	− 9·9	− 58·1	*z*	276·6	− 197·1
	9·2	56·8		275·9	196·8
	− 9·6	− 57·5		276·3	− 197·0
b	+ 21·9	+ 199·1	*b'*	302·8	+ 187·5
	22·7	199·2		303·4	186·8
	+ 22·3	+ 199·1		303·1	+ 187·2
o	47·5	+ 145·5	*w*	316·2	− 102·7
	49·0	145·6		315·8	101·2
	48·2	145·5		316·0	− 102·0
p	56·4	+ 59·6	*c*	317·6	+ 63·9
	56·9	60·6		317·2	64·3
	56·6	+ 60·1		317·4	+ 64·1
d	151·7	+ 16·9	*y*	317·2	− 216·5
	151·3	16·3		317·4	217·1
	151·5	+ 16·6		317·3	− 216·8
e	169·8	+ 99·9	*l*	328·5	− 160·5
	169·5	100·5		328·2	160·2
	169·7	+ 100·2		328·4	160·4
g	171·9	+ 192·8	*t*	345·3	− 132·2
	172·3	193·0		344·4	131·3
	172·1	+ 192·9		+ 344·9	− 131·8

Name.	X	Y	Name.	X	Y
f	171″·4	+146″·9	*v*	+386″·9	−162″·3
	170·3	146·6		387·2	161·5
	171·1	+146·8		387·0	−161·9
k	183·0	−116·7	*r*	396·6	− 72·8
	182·5	117·3		396·6	72·5
	182·8	−117·0		396·6	− 72·7
n	223·3	−183·7	*u*	398·9	−198·0
	222·8	184·1		398·9	−198·2
	223·1	−183·9		398·9	−198·1
m	223·6	−163·7	*x*	442·3	−146·7
	222·7	164·2		442·7	145·6
	223·2	−163·9		442·5	−146·2
i	232·8	+ 73·6	*s*	555·8	− 41·7
	233·1	75·0		555·8	− 42·2
	232·9	+ 74·3		+555·8	− 41·9

TABLE II.—*Comparison of Measures.*

Name.	*Wilson.* X_W	Y_W	*Rosse.* X_r	$X_r - X_W$	Y_r	$Y_r - Y_W$	*Struve.* X_s	$X_s - X_W$	Y_s	$Y_s - Y_W$	
	″	″	″	″	″	″	″	″	″	″	
q	− 94·4	− 20·6	...	...	...	...	− 97·6	−3·2	− 20·9	−0·3	
a′	− 9·6	− 57·5	− 10·9	−1·3	− 59·8	− 2·3	− 12·3	−2·7	− 57·2	+0·3	v. v. f.
b	+ 22·3	+199·1	+ 23·0	+0·7	+199·7	+ 0·6	+ 21·6	−0·7	+199·7	+0·6	
o	48·2	+145·5	46·8	−1·4	+149·5	+ 4·0	+ 49·6	+1·4	+144·0	−1·5	
p	56·6	+ 60·1	48·1	−8·5	+ 61·3	+ 1·2	+ 60·6	+4·0	+ 57·9	−2·2	Ill defined.
d	151·5	+ 16·6	154·4	+2·9	+ 17·5	+ 0·9	+152·5	+1·0	+ 12·8	−3·8	$=a$ in mag.
e	169·7	+100·2	172·8	+3·1	+ 98·6	− 1·6	+172·5	+2·8	+ 98·8	−1·4	
g	172·1	+192·9	173·9	+1·8	+193·2	+ 0·3	+174·3	+2·2	+191·5	−1·4	v. v. f.
f	171·1	+146·8	172·3	+1·2	+144·5	− 2·3	+174·6	+3·5	+142·2	−4·6	
k	182·8	−117·0	187·4	+4·6	−121·3	− 4·3	+186·8	+4·0	−121·0	−4·0	
n	223·1	−183·9	...	...	...	...	+224·9	+1·8	−184·7	−0·8	
m	223·2	−163·9	...	...	...	...	+225·0	+1·8	−164·3	−0·4	
i	232·9	+ 74·3	235·8	+2·9	+ 70·8	− 3·5	+238·2	+5·3	+ 71·5	−2·8	$>h$.
h	246·9	+103·5	246·2	−0·7	+100·5	− 3·0	+251·1	+4·2	+101·7	−1·8	v. nebulous.
z	276·3	−197·0	...	...	...	...	+279·6	+3·3	−199·7	−2·7	
b′	303·1	+187·2	303·6	+0·5	+183·4	− 3·8	+308·3	+5·2	+186·3	−0·9	
w	316·0	−102·0	323·5	+7·5	−115·0	−13·0	+317·7	+1·7	−102·0	0·0	
c	317·4	+ 64·1	325·3	+7·9	+ 63·1	− 1·0	+321·1	+3·7	+ 62·8	−1·3	
y	317·3	−216·8	...	...	...	...	+321·8	+4·5	−219·5	−2·7	
l	328·4	−160·4	...	...	...	...	+332·4	+4·0	−162·2	−1·8	
t	344·9	−131·8	...	...	...	...	+347·8	+2·9	−128·9	+2·9	Ill defined.
v	387·0	−161·9	...	...	...	...	+390·7	+3·7	−161·7	+0·2	
r	396·6	− 72·7	398·2	+1·6	− 91·6	−18·9	+396·9	+0·3	− 74·0	−1·3	
u	398·9	−198·1	...	...	...	...	+402·4	+3·5	−200·8	−2·7	
x	442·5	−146·2	...	...	...	...	+442·0	−0·5	−148·5	−2·3	v. v. f.
s	+555·8	− 41·9	...	...	...	...	+555·8	0·0	− 42·0	−0·1	

Note on the Star DM + 27°·725. By A. M. W. Downing, M.A., D.Sc.

This star was selected as one of the *Nautical Almanac* zodiacal stars on the authority of the Harvard Photometry, where it appears as No. 893 of the General Catalogue, with magnitude 6·3. The magnitude 5·9 assigned to the star in the DM is a misprint for 9·5, as is explained in *B. B.* vi. p. 378.* And as a tolerably comprehensive search failed to find the star in any modern catalogue it appears probable that the Harvard observations, assigned to this star, really refer to some other object. This view is confirmed by the evidence afforded by a photographic plate which the Astronomer Royal most kindly had taken for the express purpose of identifying the star. It was found, on examining the plate, that the object nearest the assigned place was fainter than the 9th mag. Professor Pickering has also been so good as to furnish information bearing on the subject derived from photographs taken on 1890 December 29, 1892 January 8 and December 28, and 1893 October 5. On these plates the star appears to be of about the 10th mag., and shows no evidence of variation. It may be concluded, therefore, that the mag. 9·5 assigned to this star by the Bonn observers is correct, and that the Harvard measures refer to some other object.

That the object measured at Harvard College was not the neighbouring star DM + 27°·723 appears probable from the results of measures of the latter object which Professor Pickering has kindly communicated. The mean of four measures gives 6·6 as the magnitude of this star.

The star DM + 27°·725 ought not, therefore, to have been included in the *Nautical Almanac* list of zodiacal stars (which is restricted to stars of magnitude not fainter than 6·5), and should be struck out wherever it appears in the Occultation Section of the *Nautical Almanacs* for 1896 and 1897, as well as in the Catalogue of Zodiacal Stars printed in the appendix to the last-named volume.

Nautical Almanac Office :
1894 *March* 5.

* Mr. Chandler also points this out in *Ast. Nach.* No. 3214.

On the Spectra of Certain Stars (Second communication). By the Rev. T. E. Espin, M.A.

(1) DM + 49°·41. R.A. $0^h\ 12^m{\cdot}2$. Decl. + 49° 44′ (1900). Mag. 9·0. The spectrum of this star was found to be IV. type at Harvard. As this star has not been observed visually, as far as I am aware, it was looked up Dec. 11, and found to be fine red, Spectrum IV.! The bands are very well seen. Mag. 8·6.

(2) Anonymous. R.A. $0^h\ 16^m{\cdot}8$. Decl. + 58° 36′ (1900). A star found October 8, not in Argelander. The colour is fine red, and the spectrum banded, but too faint to be sure whether it is type III. or type IV. As it was not in Argelander, and not recorded in previous sweeps, it has been watched for variation. The following are the observations of it:—

October 8, Mag. 10·0.
October 11, Mag. 9·7.
October 19, Mag. 9 7.
December 30, Mag. 9·5.

The star may therefore have brightened slightly.

(3) Anonymous. R.A. $2^h\ 10^m{\cdot}0$. Decl. + 58° 3′ (1900). Found December 11. Not in Argelander. Mag. 9 0. Pale red. Spectrum III. type. The comparison of photographs 1893 January 20, 26, December 12, shows no variation.

(4) *T Persei.* R.A. $2^h\ 12^m{\cdot}2$. Decl. + 58° 59′ (1900). As this star has not been observed with the spectroscope, it was set for on December 11, and found to be mag. 8·6. Colour red. Spectrum III. !!! The bands are large, but not deep. The spectrum is similar to many stars in this region. I have found no part of the heavens where so many red stars are congregated. If a circle with about 1° radius is drawn with centre R.A. $2^h\ 10^m$, Decl. 57°25′ (1855), it will include fourteen stars of type III.

(5) DM + 56°·731. R.A. $2^h\ 44^m{\cdot}8$. Decl. + 56° 32′ (1900). Mag. 9·5. This is a star of type V. discovered by photography at Harvard. I have observed it on several occasions, and it always appeared to have a monochromatic spectrum. On December 11 I reobserved it. There is a very faint continuous spectrum. Calling the total light of the star 10, the bright line at or near F would equal 9, the continuous spectrum 1. There is also an inequality in the spectrum, probably the usual one at 540. The bright line is remarkably vivid, and appears to project beyond the spectrum. Whether this is due to irradiation or actual projection I am unable to tell.

(6) DM + 61°·734. R.A. $4^h\ 40^m{\cdot}1$. Decl. + 61° 0′ (1900). Mag. 7·7. This star was discovered by Mr. Gage with his 15-inch reflector at Wolsingham. He informed me that it had a fine III.-type spectrum. It was observed by me 1894 Jan. 7, and found to be mag. 7·5. Colour orange red. Spectrum

III. ! ! ! The spectrum is normal; all Dunér's bands are seen. The bands at the violet end are intense, and the spectrum here is discontinuous.

(7) Var. R.A. $19^h\ 9^m\cdot6$. Decl. 49°29′ (1900). A star not in Argelander. Anncunced as variable by Deichmüller (*A.N.* 3191). Observed December 14 in moonlight as 8·8 mag. Colour red. Spectrum III. ! ! Faint in moonlight. The bands seem to be deep in the violet end, and the spectrum is here discontinuous. When the spectrum is brought to a line, it is beaded with bright dots.

(8) DM + 24°·3902. R.A. $19^h\ 46^m\cdot3$. Decl. + 24°41′ (1900). Mag. 9·2. First seen October 11. Mag. 8·7. Very red; banded spectrum. I think III. November 13, 8·9 mag.

(9) χ *Cygni.* R.A. $19^h\ 46^m\cdot7$. Decl. + 32° 40′ (1900). August 12, mag. 9·0. Perfectly discontinuous III.-type spectrum. There seems to be nothing but a few bright isolated bands. Colour very red. August 14, mag 9·1; very red. The spectrum quite discontinuous; one bright band stands out near centre of the spectrum far brighter than the rest; others are, however, seen. Allowing for the faintness of the light, the spectrum seems similar to what it is at maximum.

(10) Var. R.A. $20^h\ 47^m\cdot4$. Decl. + 45° 50′ (1900). An anonymous star, announced as variable by Wolf. December 9, mag. 9·0; red. There are bands, and I think they are those of type III.

(11) *Es. Birm.* 696 = DM + 45°·3349. R.A. $20^h\ 54^m\cdot5$. Decl. + 46° 5′ (1900). Mag. 8·1. My observations of the spectrum of this star differ strangely. 1888 May 4 I noted that the blue and violet are singularly faint, and the spectrum very unpronounced for so finely coloured a star, sometimes thought to be III., sometimes IV. May 9, III.! 1891 October 2, III. ! ! ! November 26, III. 1893 September 9, III. ! ! There is no doubt about the spectrum; the bands are large and faint. The photographs suggest unsteadiness of light. My observations of the magnitude are—

1885 December 15, 25, 8·0 mag.
1886 January 15, June 30, August 20, 8·0 mag.
1891 October 2, 8·5 mag.
1891 November 26, 8·3 mag.
1893 September 9, 7·6 mag.
1893 September 11, 8·0 mag.
1893 September 12, 8·2 mag.
1893 September 14, 8·1 mag.
1893 September 16, 8·2 mag.
1893 September 19, 8·2 mag.
1893 September 27, 8·3 mag.
1893 November 13, 8·2 mag.

The variation seems very doubtful.

(12) DM + 57°·2487. R.A. $22^h\ 10^m{\cdot}5$. Decl. + 57° 31′ (1900). Mag. 9·1. October 6, mag. 9·0; very red; the bands are very large, and at first it was thought to be IV., but afterwards I was convinced that it was III. ! ! ! The spectrum is quite discontinuous, and some of the bands are of unusual size.

Some Nebulous Objects not in the New General Catalogue of Nebulæ. By the Rev. T. E. Espin, M.A.

While sweeping for red stars I have encountered various nebulous objects. These have been carefully marked on the chart of Argelander of that region, estimating their position from the neighbouring stars. Most of these were subsequently found in the New G.C. of Nebulæ. The following are, however, new. In several cases they are partially resolved, and may really be gatherings of very faint stars. The places which are approximate only, but which are probably correct to within 10^s of R.A. and 3′ of Decl., have been brought up to 1860.

(1) R.A. $19^h\ 16^m\ 33^s$ Decl. + 20° 29′. Probably a cluster of very faint stars. October 8.
(2) R.A. $19^h\ 30^m\ 46^s$ Decl. + 40° 44′. Faint nebulosity. October 6.
(3) R.A. $19^h\ 33^m\ 8^s$ Decl. + 19° 54′. Very faint, extending N.P. from a 9·5 mag. star. October 8.
(4) R.A. $19^h\ 36^m\ 43^s$ Decl. + 37° 19′. Nebula round a group of faint stars. September 16.
(5) R.A. $19^h\ 37^m\ 5^s$ Decl. + 27° 10′. While sweeping on September 14 I suddenly came upon a dark space. On carefully examining the field there is evidently a large faint nebulosity, mixed up with stars, sharply defined on *f* side, stretching N. and S. Max Wolf's photograph shows this nebulosity.
(6) R.A. $20^h\ 4^m\ 43^s$ Decl. + 34° 33′. Faint nebulosity. September 19.
(7) R.A. $20^h\ 6^m\ 0^s$ Decl. + 40° 46′. Extremely faint nebula within a circle of bright stars. October 6.
(8) R.A. $20^h\ 11^m\ 42^s$ Decl. + 24° 45′. Large, misty patch, partly resolved. October 8.
(9) R.A. $21^h\ 5^m\ 39^s$ Decl. + 46° 17′. Faint, extending N. from DM + 46°·3214, 9·4 mag. September 9.
(10) R.A. $21^h\ 7^m\ 12^s$ Decl. + 47° 11′. Faint, large, many small stars. September 9.
(11) R.A. $21^h\ 18^m\ 44^s$ Decl. + 54° 51′. Faint, diffused, some faint stars. September 16.
(12) R.A. $21^h\ 39^m\ 19^s$ Decl. + 52° 18′. Misty patch, partly resolved. September 16.

(13) R.A. $21^h\ 39^m\ 59^s$ Decl. + 52° 37′. Faint, partly resolved; stars 14 mag. September 16.

(14) R.A. $22^h\ 5^m\ 21^s$ Decl. + 52° 8′. A remarkable cluster with six distinct radiating branches. Stars from 12 to 15. Very beautiful. September 19.

(15) R.A. $22^h\ 11^m\ 11^s$ Decl. + 53° 21′. Haze round some dozen faint stars.

Description of a Perfectly Achromatic Refractor.

By H. Dennis Taylor.

Now that photography is becoming such an indispensable supplement to eye observation in astronomical work, the need for a form of refracting telescope which is perfectly corrected for photographic as well as for visual purposes becomes more and more apparent. And if, simultaneously with that condition, a great improvement in the character of the visual image can also be obtained, then we have the elements of a very substantial improvement in the refractor. In a paper which I had the honour of reading to this Society in November last I pointed out the considerable loss of light for defining purposes which must take place, owing to the usual colour aberrations ever present in the case of double object glasses made of ordinary crown and flint glasses. I there gave the losses of light, as determined by a theoretical method, for certain objectives of various sizes. Since then I have been able to carry out a delicate experiment with a 12½-inch object glass, whereby the amount of light lost owing to the colour aberrations was separated from the real image, and rendered approximately measurable. The result certainly confirmed the figures giving percentage of light lost, which I had previously arrived at by an *à priori* line of reasoning. I hope to have the pleasure of describing this experiment in a future paper. Thus I feel justified in saying that the principal improvement in the visual image to be expected from the almost perfect achromatism attained in this new objective is a more brilliantly defined image, the fine details being rendered in more black and white contrast than we have been accustomed to see, thus standing out, in an artistic sense, with greater sharpness. For this new objective is to all intents and purposes as achromatic as a reflecting telescope. My justification for this statement is as follows:—

It must be remembered that in practice the reflecting *telescope*, mirror and Huyghenian eye-piece combined, is not absolutely achromatic. However absolute may be the achromatism of the primary image, yet the eye-piece, if of the Huyghenian or Ramsden type, always introduces its own colour aberrations. Taking the case of a Huyghenian eye-piece of 1 inch equivalent

focal length, the virtual longitudinal colour aberration from B to Hγ (G′) which it would introduce would amount to about three-hundredths of an inch. This amount in the case of a reflector of a focal length equal to seven and a half times its aperture is about equal, as regards detriment to vision, to a longitudinal colour aberration of six-hundredths of an inch in the case of either a reflector or a refractor of a focal length equal to fifteen times its aperture. I shall shortly point out that in the case of even an objective of 2 feet aperture, and of 36 feet focal length, of the new construction, the residual longitudinal colour aberrations need not be expected to amount to as much as six-hundredths of an inch; while at the same time an achromatic single lens eye-piece may be employed, presenting no spectrum and having only two reflecting surfaces. Or, supposing a 1-inch Huyghenian eye-piece to be generally employed on such a telescope, the objective may be sufficiently over-corrected to counteract the longitudinal colour aberrations of that eye-piece.

The problem of devising a perfectly achromatic objectglass has occupied the attention of many scientific men and opticians for a considerable time. The Rev. Vernon Harcourt spent twenty-five years or so of his life in melting and trying new forms of glass with a view to producing two glasses of similar rationality. But none of his experiments resulted in practical success. The problem has also deeply occupied the attention of Sir G. G. Stokes and the late Professor Pritchard. In short, it is a problem which has exerted a great fascination over many minds. Some, notably Professor Hastings in America, and Professor Abbe of Jena, fully realised that practical success need not by any means depend upon the employment of only two lenses in the objective. Professor Hastings published an article in the *American Journal of Science and Art* (for December 1879) in which he gave the results of calculations of several forms of objectives made of three kinds of glass, in some of which a remarkably near approach to perfect achromatism seemed to be possible. However, none of these combinations seem to have been worked into practical shape, and, so far as I know, principally because one or other of the glasses employed—all, I believe, the production of Mons. Feil of Paris—could not be made hard enough or durable enough for practical use.

In 1884 the justly celebrated firm of Herren Schott und Ges started their optical glass manufactory at Jena for the principal purpose of making both old and new varieties of optical glass on a commercial scale, at the same time addressing themselves in a most thorough and scientific manner to the problem of producing two glasses whose rationality of dispersion should be identical, at the same time that the difference between their dispersive powers should be sufficient to enable a perfectly achromatic double object glass to be constructed, without having to resort to impracticably deep curves.

But in spite of the skill and scientific method brought to bear upon this problem, and exhaustive trials of all known substances capable of being fused into glasses, it cannot be said that that aim has been realised; and I really believe that a practicable *double* object glass, even reasonably free from secondary colour aberration, is as far from being realised as it was ten years ago. For it should be borne in mind, when aiming at an object glass of two glasses which shall be perfectly achromatic, that although it may be possible to considerably reduce the discordance between their respective rationalities of dispersion, yet the reduction of secondary spectrum at the focus accruing from that improvement may be largely counteracted or even wholly neutralised by the deeper curvatures or higher powers of the two component lenses, which are rendered necessary by that smaller difference of dispersive powers of the two glasses which has hitherto inevitably accompanied the closer agreement in rationality of dispersion. In short, whether using two or three lenses, the deeper the curves, or the more powerful the component lenses, required for obtaining a given focal length, the more close and strict must be the concordance of rationality of dispersion between the glass (or two glasses combined) forming the positive element and the glass forming the negative element of the objective. To be sure, certain double combinations have been made in which the secondary spectrum has been reduced to about half the usual amount. For instance, Dr. Czapski of Jena * describes a form of objective of which two or three were made by Bamberg of Berlin, and also, I believe, by Dr. Czapski himself, in which the positive lens was made of Schott's dense barium phosphate crown S ·30, and the negative lens of Schott's borate flint S ·8 or S ·7. This is somewhat similar to a combination of the same flint with a nearly similar crown which was carried out by Messrs. Cooke about seven years ago, and found to make a more than usually achromatic object glass. But the borate flints are too soft for practical purposes, and, moreover, Dr. Schott has informed us that it is almost impossible to get good discs of this glass of over 5 inches aperture.

A few years ago Professor Hastings patented in America another form of double object glass in which the secondary spectrum was reduced to about half. For his positive lens he made use of potassium silicate crown (Schott's 0·13), and for his negative lens Schott's boro-silicate flint glass 0·161. This combination turned out to be impracticable: the crown is too hygroscopic, and the flint, Dr. Schott has informed us, cannot be made good enough.

Having considered all the facts of the case, I was inevitably

* In Dr. Czapski's work, *Theorie der optischen Instrumenten (nach Abbe)*, on page 131, will be found graphic diagrams showing the somewhat reduced amounts of secondary spectrum exhibited by these combinations as compared with that exhibited by an ordinary object glass of crown and dense flint.

led, as a few other inquirers have been led, to the conclusion that there was little prospect of making an object glass of two lenses whose achromatism would be so much superior to that of the ordinary objective as to fully compensate for the other practical disadvantages which would ensue, either in the shape of curves difficult to work or a confined field of view, not to mention several other serious drawbacks.

Some two years ago I entered into an investigation as to the possibility of producing a triple object glass, free from secondary spectrum, whose curves should be such as to permit of all the other necessary or desirable optical qualities and certain practical advantages being attained. After a very searching and prolonged trial of all the likely combinations of glasses picked out of the numerous new and old types of glass, put at the disposal of the optical world by the genius of Dr. Schott and Professor Abbe, and by help of every opportunity for investigation and experiment afforded me in the workshops of Messrs. Cooke, I at last realised a triple combination of three different sorts of glass, which, while yielding a degree of achromatism almost greater than anything which I had at first ventured to hope for, at the same time rendered possible the achievement of certain other important optical qualities and practical conveniences which yet remain to be specified.

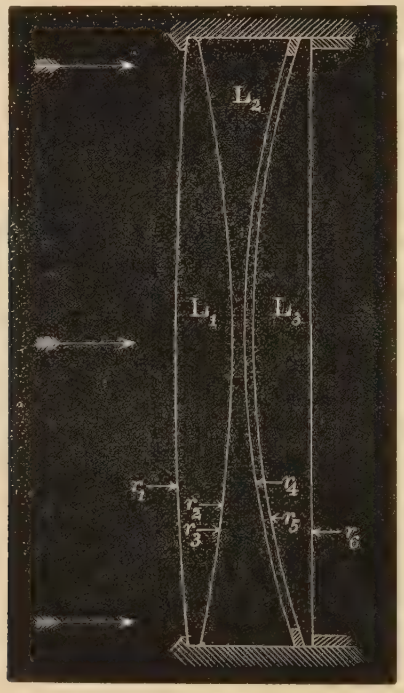
Taking the optical qualities first—

1. This objective can be made free from spherical aberration for all colours simultaneously. This is of the utmost importance in the case of an objective having any pretensions to perfect achromatism. In a double objective of ordinary construction having a double concave or concavo-plane flint lens, it it well known that when the brightest yellow green rays are perfectly corrected for spherical aberration, then the red rays are under-corrected, and the blue rays are over-corrected for spherical aberration ; a condition of things, however, which escapes notice in the presence of the greater fault of the secondary spectrum.

2. The objective is so arranged as to give the largest possible field of good definition. The image of a star formed at, say, 2° from the optic axis is free from coma or side flare, and also from colour. If a star is focussed into the centre of the eye-piece and the objective is tilted, even very considerably, then the image of the star remains symmetrical and without side flare, and only showing the inevitable astigmatism. This condition is of the utmost importance in the case of an objective when used for photographing celestial objects having considerable angular extension, or for transit instruments which are generally provided with eye-pieces having a certain amount of lateral travel.

3. The curves are also such that a ray parallel to the optic axis traced through the margin of the objective enters and leaves the flint lens at approximately equal angles. This is of con-

siderable importance in the case of deep curves, and especially in large-sized objectives, because the flint lens is by a long way the weakest, in a mechanical sense, of the three lenses, and therefore most liable to flexure across its diameter, and such flexure would in the large sizes, and under ordinary circumstances, give rise to considerable aberrations of the marginal rays from true focus. But if refraction of such marginal rays takes place about *equally* at both surfaces, then the optical results of such flexure or sagging of this lens are reduced to practically nothing. The action of the edge of the flint lens upon the marginal rays is in this case the same as that of a prism when set at minimum deviation, which may be rotated through a relatively considerable angle without bringing about a very perceptible increase in deviation.



The accompanying diagram is a section of the objective taken along a diameter. The focal length is eighteen times the aperture. If specially required, the aperture can be made equal to one-fifteenth of the focal length in the case of sizes up to 8 or 10 inches aperture. A relatively large focal length is, however, most to be recommended.

The lens L_1 which is placed first to receive the parallel rays

is made of Schott's baryta light flint glass 0·543, whose refractive index for D ray is about 1·564.

The negative lens L_2 is made of Schott's new boro-silicate flint glass, which is a variety of their 0·164. Its refractive index for D is 1·547.

The glass of which the third lens L_3 is made is a sort of light silicate crown (Schott's 0·374), which differs from ordinary hard crown in having a lower dispersive power and a somewhat different rationality. Its index of refraction for D is about 1·511.

The separation between the negative lens L_2 and the positive lens L_3 is designed for the purpose of correcting the spherical aberration perfectly for all colours.

The contiguous second and third surfaces r_2 and r_3 are curved to exactly the same radius; also the fourth and fifth surfaces r_4 and r_5 are curved to the same radius. The last surface r_6 is concave, and curved to a radius equal to nearly twice the focal length. There are thus three hollow surfaces whose figuring can be directly tested by reflected light, while each of the other three convex surfaces can be tested for figuring in an indirect manner. For instance, if all three concave surfaces are approved, and faults at the focus are found to exist which must be caused by defective figuring in one or more of the convex surfaces, then they can be tested in the following manner:—

If the faults at the focus are due to defective figuring in the second surface, those faults will more or less totally disappear on introducing some liquid having a refractive index approximately equal to that of glass between the two surfaces r_2 and r_3. If it is desired to test the fifth surface, then the brass ring separating L_2 from L_3 is taken out and put between the second and third surfaces, while L_2 and L_3 are brought close together. The objective being first tested in this state with its surfaces dry and the faults at the focus noted, then the refracting liquid may be introduced between the fourth and fifth surfaces, when, if the fifth surface is faulty in figure, the faults at the focus will more or less disappear. But if both the second and fifth surfaces are not found to be faulty by this method, then the conclusion is that the first surface r_1 is really at fault. Thus, all surfaces may be easily tested either directly or indirectly, and the whereabouts of small errors of figuring revealed, which are too fine to be appreciated by the use of spherometers or other mechanical tests. Thus no time need be lost in refiguring the wrong surfaces.

The baryta light flint and light silicate crown glasses used for the two exterior lenses are as hard and durable as ordinary crown glass, and if anything more colourless and transparent.

The boro-silicate flint glass used for the negative lens is of quite a different type from that used by Professor Hastings for his patented doublet. Indeed, this flint is the only available

glass which can be used for apochromatic objectives, and can be turned out in large-sized discs. Its mechanical hardness is very great—greater indeed than that of hard crown. It is beautifully transparent to all the brighter rays of the spectrum, but begins to absorb the violet rays after the G line is passed. This characteristic imparts a lemon yellow colour to a lens when of two inches thick or over, but it is scarcely noticeable in the case of lenses of moderate sizes.

In a town atmosphere, where sulphuretted hydrogen or sulphuric acid abounds, it would not be safe to use this flint for an exterior lens; but when enclosed between the two positive lenses, as in this objective, we have no reason for expecting it to be any less permanent in its polish than the dense flint glass which has hitherto been used for double object glasses of the usual construction. Or even supposing a slight amount of tarnishing of its surfaces did take place after many years, as is the case with the flint glasses hitherto used for telescope object glasses and prisms, it may yet be asked whether the transparency or light-transmitting power of the objective would be in any degree impaired. We know well enough that a tarnished optical surface does not look so brilliant *by reflected light*. It may seem a somewhat startling statement to make, but nevertheless it is a fact, that certain flint glasses which we have experimented with by local tarnishing have been found to transmit actually more light where tarnished than where the surfaces had had their original polish preserved, and in no case has it ever been found that tarnish in any perceptible degree interfered with transparency.* We have yet to make some further experiments in this direction, and I hope at some future time to throw the proofs of this statement on the screen for your inspection. Those who use ordinary refractors may congratulate themselves on the fact that the light-gathering power of their objectives slightly increases with age, provided the surfaces are kept clean. However, leaving that interesting point for the present, I may say that enough experiments, extending over about a year and a half, have been made with this new glass to fully satisfy Messrs. T. Cooke & Sons of its lasting properties, and to warrant them in guaranteeing the permanency of this objective.

I will here give a table setting forth the optical properties of the three glasses, and also showing how very closely all the rays of the spectrum can be refracted to one focus.

* Of course this statement is not intended to apply to the case of a surface which is actually corroded, so as to appear at all *grey* or *milky* by reflected light.

1	2	3	4	5	6
Region of Spectrum.	Partial Dispersions or $\Delta\mu_1$ for Glass 0·543.	Partial Dispersions or $\Delta\mu_3$ for Glass 0·374.	Combined Partial Dispersions or $\Delta\mu_1+\Delta\mu_3$ for Glasses 0·543 and 0·374.	Proportional Dispersions C to F being Unity for Glasses 0·543+0·374.	Proportional Dispersions for Boro-silicate Flint 0·658.
C to F	·01115	·00844	·01959	1·0000	1·0000
...	(1·0000)	(1·0000)			
A to C	·00374	·00296	·00670	·3420	·3425
...	(0·3354)	(0·3507)			
D to F	·00790	·00593	·01383	·7059	·7052
...	(0·7085)	(0·7026)			
E to F*	·00369	·00274	·00643	·3282	·3278
...	(0·3309)	(0·3247)			
F to G_g(Hγ)	·00650	·00479	·01129	·5763	·5767
...	(0·5830)	(0·5675)			
F to H_1*	·01322	·00976	·02298	1·1730	1·1745
...	(1·1857)	(1·1564)			

This table is calculated on the supposition that

$$\frac{1}{\rho_1}=\frac{1}{\rho_3},$$

where $\frac{1}{\rho_1}$ = the sum of the reciprocals of the radii of the first lens and $\frac{1}{\rho_3}$ = the sum of the reciprocals of the radii of the third lens. This pretty well represents an average case, and with the present meltings of glass gives the most perfect colour correction. The curves may of course be varied within certain limits in order to compensate for variations in the optical qualities of the glasses.

But under the conditions above defined it is evident to those conversant with optics that the partial dispersions for these two positive lenses combined, for any given region of the spectrum, is obtained by simply adding together their respective dispersions for that region of the spectrum.

In column 2 are given the partial dispersions of baryta light flint glass 0·543 for the different regions of the spectrum indicated in column 1.

In column 3 are given the corresponding partial dispersions for the light silicate crown glass 0·374. The figures in brackets beneath these in columns 2 and 3 indicate the relative partial dispersions for each region of the spectrum when the dispersion from C to F for each glass is taken as unity.

* Figures relating to spectral regions E to F and F to H_1 are calculated from Messrs. Schott's measurements by means of Cauchy's dispersion formula.

In column 4 are given the sums of the partial dispersions for each region of the spectrum, obtained by simply adding together the partial dispersions for 0·543 and 0·374 respectively for each region of the spectrum indicated in column 1.

In column 5 the combined dispersion for the two positive lenses for the region C to F is taken as unity, and the relative proportions of the combined dispersions for the other regions of the spectrum are expressed in fractional parts of the combined dispersion from C to F. Finally, in column 6 are given the corresponding proportional dispersions in the same sense as in column 5 for the boro-silicate flint glass 0·658. I need scarcely explain that the perfection of the achromatism of the triple combination is made manifest by the remarkably close agreement between the proportional dispersions for the two positive lenses combined, on the one hand, and the proportional dispersions for the negative lens on the other hand.

The rationality of dispersion, as it is generally termed, is in remarkably close agreement.

These figures do not pretend to accuracy in the fourth decimal place, but I would point out that an error of ·0010 in any one of these figures would, in the case of a 6-inch O.G. of 108 inches focal length, correspond to a longitudinal aberration of only ·02 inch; an amount which, I believe, the most delicate test could scarcely reveal, owing to the cone of rays close to focus merging in such a remarkable manner into the cylindrical form.

Owing to the expensive nature of experiments of this sort, the first objectives actually made on this principle and finished were of the moderate aperture of $3\frac{1}{2}$ inches; but these were found so successful as to leave no doubt about much larger sizes being practicable should the glass be forthcoming. However, a recent visit to Dr. Schott and Professor Abbe at Jena has quite set at rest any doubts as to the possibility of turning out discs of considerable sizes, many discs of moderate dimensions, up to 9 inches, and possibly 12 inches diameter, being already available. Dr. Schott is of opinion that only actual experience with future meltings will determine up to what sizes it will be possible to manufacture perfect discs.

It is easily seen from the diagram that the total thickness of glass in this new objective is a trifle over $1\frac{1}{2}$ times the thickness of glass used in an ordinary double objective of the same aperture, and it might be thought that a serious loss of light might result therefrom in large sizes. I have recently made some experiments upon the transparency of some blocks from 4 to 5 inches thick of the three glasses used in the objective, and have been pleased to find that a thickness of 5 inches of the baryta light flint absorbed only 40 per cent. of candle light; a thickness of $4\frac{7}{8}$ inches of boro-silicate flint absorbed 27 per cent. of candle light (though doubtless more of daylight); while a

thickness of $5\frac{1}{16}$ inches of the light silicate crown absorbs only 20 per cent. of candle light.

These figures must be regarded as preliminary. I intend to check the results and present them in a more accurate and digested form on a future occasion. But as far as they go these results are surprising as showing what an improvement has taken place in the transparency of glasses (a few blocks of ordinary crown and flint glasses were also tried and found nearly as transparent as the block of 0·374) within the last few years, when thicknesses of four inches only were generally found to absorb as much as one half of the light.

Thus this triple objective might be made in very large sizes, 2 feet aperture or so, before the loss of light due to thickness would become so great as to put it on an equality as regards light-gathering power with a Newtonian reflector of average condition and similar aperture.

Buckingham Works, York.

Brilliant Detonating Fireball of 1894 *January* 25.

By W. F. Denning.

A very murky condition of sky, with occasional rain, appears to have prevailed over a considerable part of England on the night of 1894 January 25, and the only celestial objects visible were *Jupiter* and two or three of the brighter stars shining very dimly through the clouds. At about $10^h\ 1^m$, in the Midland Counties, the dark atmosphere was instantaneously illuminated with a light perfectly dazzling in its intensity, and people who noticed it thought for a moment that the full Moon had suddenly come into view. On glancing upwards, however, they immediately detected the real cause in the form of a large pear-shaped fireball rolling across the sky from the direction (as observed at many places) of N.W. to S.E., and scattering behind it a bright tail of sparks. Those who obtained the most complete view of the object describe it as small at first, like an ordinary shooting star, but, after traversing about one-third of its course, it appears to have suddenly burst out into a startling size and brightness, and afterwards divided into two parts. As it disappeared the section of its track following the nucleus was beaded with fragments decreasing in size from the foremost. The colour of the latter was variously described as blue and green, and several observers compare it with the electric light; the tail and hinder parts of the nucleus appear to have been yellow merging into red. From one to four minutes after the meteor had disappeared alarming detonations were heard at Worcester, Droitwich, Birmingham, Alvechurch, Malvern, Ross, Stroud, Cheltenham, and numerous other places in the same

district. Houses were shaken, windows and furniture were rattled violently, and there was a perceptible vibration which many people quite naturally mistook for an earthquake shock.

I have collected about forty-five descriptions of the fireball, and though the majority of these are merely of a popular character, containing few details useful for scientific purposes, there are several by astronomical observers, and these enable a satisfactory path to be derived. It may be interesting to quote from some of the best accounts:—

Clapham, near Lancaster.—Very brilliant meteor seen at $10^h\ 5^m$, January 25. The sky was covered with light clouds, so that only the larger stars could be seen through the film. When first observed the meteor was near the *Pleiades*, its course being close to the belt of *Orion*, and it disappeared in the neighbourhood of η *Canis Majoris*. Duration six seconds.—*T. R. Clapham.*

King's Heath, Birmingham.—At $10^h\ 3^m$, the sky being so hazy that only *Jupiter* and one or two stars were visible, a brilliant light suddenly appeared in the N.W. sky. On looking up a fireball of extraordinary brilliancy was perceived. Its motion was remarkably slow, and a train of great length followed in its wake. It traversed an arc of 120°, passing between *Jupiter* and the *Pleiades* and breaking up near *Rigel*. The nucleus was a brilliant bluish white colour, and its motion was uneven, as though the body encountered impediments in its flight through the atmosphere. The meteor sent out a distinct hissing sound during a considerable part of its passage. The end point was very fairly determined, for the moment after the meteor had expired *Rigel* appeared to view, and gave the transient impression of being a portion of the nucleus that had suddenly become stationary. The duration of the whole apparition was twelve seconds. About three minutes after the whole meteor burst the report came: it resembled the "boom" of distant artillery. The report was distinctly a double one, followed by a dull reverberation that resounded from all parts of the sky.—*D. E. Packer.*

Edgbaston, Birmingham.—At $10^h\ 1^m$ a detonating meteor equal in size to a half-moon was seen. The sky was overcast, only the planet *Jupiter* being dimly visible, but the meteor illuminated the country like bright moonlight. At the end of its flight the nucleus burst, showering down fragments. An audible sound was heard at $10^h\ 4^m\ 5^s$, resembling the subdued report from a piece of ordnance. The colour of the forward part of the nucleus was electric blue, the rest red; duration of flight 12 seconds. Path from Az. 15° W. of N., alt. 20°, to Az. 30° W. of S., alt. 25°.—*W. H. Wood.*

Sunderland.—Bright meteor, $= \mathfrak{U}$, and apparently directed from the place of that planet, disappeared behind a house at a

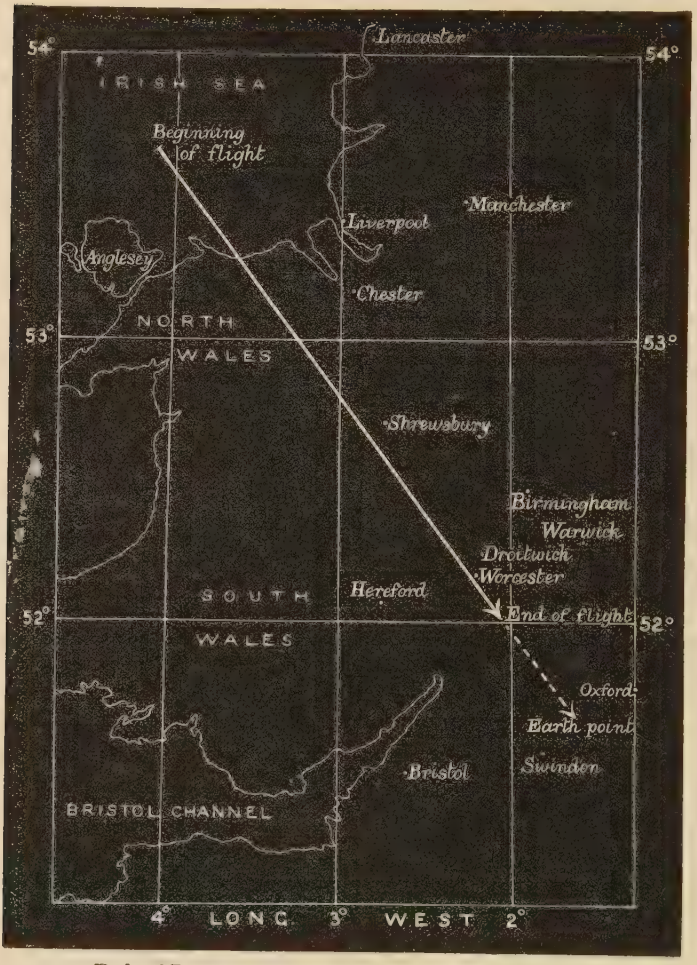
position occupied on February 2 at $9^h\ 27^m$ by $75° - 25°$. Course seen about 7°. Very slow, 1·2 second; colour green. Seen imperfectly through clouds.—*T. W. Backhouse.*

Bristol.—At $10^h\ 1^m$ a sudden and great brightening of the sky caused the observer to look upward, and the fireball was well seen during the last ten degrees of its course. The position was determined as from $242° + 59°$ to $229° + 53°$, and this may be considered pretty correct as to direction, though to include the whole visible path the line of flight must be extended back to about *η Cephei*. The place of the end-point corresponds approximately with Az. 26° E. of N., alt. 20°, and from several careful descriptions by other observers at Bristol the beginning occurred at Az. 22° W. of N., alt. 26°. Ten degrees from the point of its extinction the fireball consisted of a double nucleus, the foremost of the two parts being largest and brightest. Near the end these subdivided and formed a string of particles, when the whole phenomenon collapsed, and the sky resumed its previous darkness. The estimated duration was five seconds from the time of the first outburst of light, but the 10° of the observed path occupied less than two seconds.—*W. F. D.*

The other accounts need not be particularly referred to, as for the most part they simply describe the astonishing brilliancy of the meteor and its rough position. An observer at Henbury says it made a hissing noise as it passed, thus confirming Mr. Packer's impression.

As to the detonation that followed the meteor's disruption, this appears to have varied according to locality. Mr. W. H. Lloyd, of Stroud, describes it as a series of explosions (apparently from the N.E.) which sounded like a number of fieldpieces fired in rapid succession, followed by a volley of musketry. Mr. J. T. Southall, of Ross, Hereford, says a loud rumbling noise like an explosion was heard. It lasted two or three seconds, and two slight shocks, supposed to be due to an earthquake, were experienced. At Worcester the path of the meteor was from N.W., and shortly after its disappearance three detonations were heard, the last being of exceptional violence, shaking buildings and causing the earth to vibrate. At Droitwich it was supposed the Evesham Gas Works, twelve miles distant S.E., had blown up. At Alvechurch a detonation was heard similar to a clap of thunder. At Malvern, at $10^h\ 3^m$, a meteor of extraordinary size and brilliancy came from a N.W. direction, and three minutes after its disappearance a heavy rumbling noise was heard, immediately followed by the sound of a tremendous explosion. Mr. W. E. Clarke, of Elkstone (between Cheltenham and Cirencester), says he heard several explosions, and these seemed to come right from the opposite direction to the meteor's flight, and about ten or fifteen seconds after its disappearance. He refers to a peculiar "whirring" noise which followed the detonations, and says the nearest resemblance to it he can suggest is "the sound

made by rapidly swinging round and round a piece of tin attached to the end of a string."



Path of Detonating Fireball, 1894 January, $25^{d}\ 10^{h}\ 1^{m}$.

I have compared the various observations of the apparent path, and find there is a tolerably good agreement amongst them. The following are the resulting positions and heights:—

Observed beginning of flight over a point in lat. 53° 42′ N., long. 4° 7′ W. (24 miles N.N.E. of Amlwch, Anglesey); height 89 miles.

Observed ending of flight over lat. 52° 2′ N., long. 2° 5′ W. (Ashchurch, Tewkesbury); height 16 miles.

Earth point in lat. 51° 42′ N., long. 1° 42′ W. (Lechlade, on the eastern border of Gloucestershire).

Real length of observed path = 160 miles.

Duration of flight (mean of twelve estimates) = 9 seconds.

Velocity 18 miles per second.

Radiant point 331° + 55°, near ε *Cephei*.

The meteor passed almost vertically over Shrewsbury and Worcester, at elevations of 45 and 22 miles respectively.

The observed beginning is that noted by Mr. Clapham, whose position was 22 miles N.E. of Lancaster. Possibly the fireball may have acquired visible incandescence much earlier than this in its flight, and when it was passing over the Isle of Man, but we have no positive evidence on this point. Nearly all the observations cluster round the counties of Warwick, Worcester, Hereford, and Gloucester, and there is a singular lack of accounts from North Wales and the northern parts of England. The cloudy state of the sky at the time is no doubt in a great measure responsible for this. There is, it is true, one report from Mold, North Wales, where the meteor appears to have looked larger than the Moon, but no particulars are given as to the path.

The height at end is well indicated by the King's Heath (Birmingham) and Bristol observation. It agrees also with the interval between the explosion and detonation, as recorded at Birmingham and some other places. Mr. Packer, at King's Heath, puts the interval at 3 minutes, while Mr. Wood, at Birmingham, estimated it as 3 minutes 4 seconds. The meteor was 36 miles distant from Birmingham at the end point, and sound, travelling at the rate of 1,090 feet per second, would occupy 3 minutes in traversing 37 miles. The agreement is not so precise when the same comparison is applied to certain other stations, but it must be remembered that the time intervals were in nearly every case little better than mere guesses, and not well-determined periods. Moreover, the detonation may not have been caused by the disruption of the meteor precisely at the place of its extinction, for disintegration had been obviously going on some time before. Several of the accounts would appear to suggest that one of the explosions must have occurred when the meteoric *débris* had gone a few miles beyond the place of its disappearance in a prominently luminous state.

It is evident that distance had a great effect upon the apparent brightness of the object. At Sunderland the observer estimated it as equal to *Jupiter*, but his position was about 175 miles from the section of the meteor's path, S.E. of Shrewsbury, which he observed. At Clapham, Lancaster, the distance from the meteor was also considerable, varying from 120 to 157 miles, and here the light was thought to equal *Venus*. But at Worcester, where the minimum distance was only 22 miles, the meteor illuminated the sky with an effect exceeding that of the full Moon, and it was compared to that of the great electric search-light by several observers.

The duration of the meteor's flight was variously estimated at from 1 or 2 seconds to 1 or 2 minutes. A mean of twelve of the most reliable accounts gives 9 seconds, and from this I derive the velocity as 18 miles per second. Atmospheric resistance no doubt had the effect of considerably moderating the native speed of the meteor.

The radiant point at 331° + 55° near ϵ *Cephei* is rather a novel one for the season, and I believe the only January shower of Cepheids hitherto observed is that seen at 329° + 60°, 1886 January 4–8, by the writer at Bristol. In the summer and autumnal months there are many radiants in this region of *Cepheus*, and the position represents one of the apparently long-enduring, fixed showers which are so difficult of explanation (see *Monthly Notices*, vol. l. pp. 416 and 467).

The great detonating fireball seen in the north of England on the morning of 1879 February 24 had a radiant approximately determined at 310°+55°, and suspiciously near that of the similar fireball of January 25 last.

It is, perhaps, a little questionable whether large fireballs of the detonating class are always indicative of ordinary meteor showers, as they often appear in an isolated manner and are frequently directed from radiants situated in the vicinity of the horizon and in the western half of the sky. Slowness of motion is also a characteristic feature of these bodies. After seeing one of them an observer may watch the heavens during the remaining part of the night and note many meteors without detecting one from the fireball radiant. This has sometimes been my own experience, and there seems little reason to doubt the comparative exclusiveness of such phenomena. But of course if observations were undertaken on a wider basis, and many observers participated in them, the solitary nature of these bodies might not be confirmed. It is one of those features on which we cannot speak with certainty owing to the inadequacy of past observation. But in whatever light we regard these brilliant slow-moving fireballs it is certain they form the most interesting and attractive type of meteoric apparitions.

Bristol:
1894 *February* 4.

Erratum in Col. Markwick's paper. (*"Monthly Notices," vol. liv. No.* 3.)

In the diagram on page 139 of the "Light Curve of W Sagittarii" the numbers denoting the brightness should be shifted upwards one line, so that the top line reads 4·8 and the lowest line 5·8.

CONTENTS.

	PAGE
Fellows elected	285
Candidates proposed	*ib.*
Presents announced	286
Simon Newcomb, Remarks upon Mr. Stone's Proposed Corrections to the Measure of Time since 1864	*ib.*
E. J. Stone, Note on Professor Newcomb's Remarks	288
R. T. A. Innes, Tables to Facilitate the Application of Gauss's Method of Computing Secular Variations	289
A. Stanley Williams, On the Rotation of *Saturn*	297
Royal Observatory, Greenwich, Results of Micrometer Measures of Double Stars with the $12\frac{3}{4}$-inch Merz Refractor, made during the the year 1893	314
S. Glasenapp, New Orbit of 9 Argûs, β 101	318
A. A. Rambaut and W. E. Wilson, On the Proper Motion of Stars in the Dumbbell Nebula	319
A. M. W. Downing, Note on the Star DM + 27°.725	324
Rev. T. E. Espin, On the Spectra of Certain Stars (second communication)	325
Rev. T. E. Espin, Some Nebulous Objects not in the New General Catalogue of Nebulæ	327
H. Dennis Taylor, Description of a Perfectly Achromatic Refractor	328
W. F. Denning, Brilliant Detonating Fireball of 1894 January 25	337
Erratum	342

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